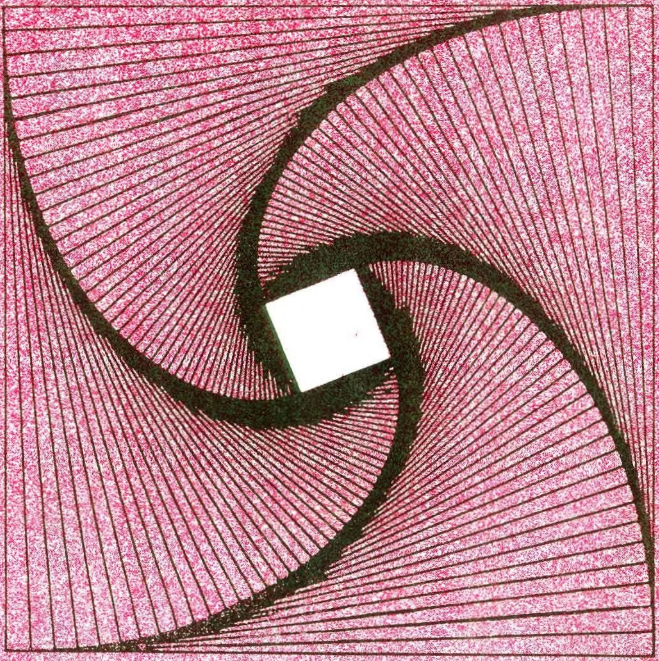
ISSN: 0972-2440

THE MATHEMATICS TEACHER



Volume: 37 Issue: 3 & 4 Year: 2001

गणितं मूर्धनि स्थितम्

AMTI

OFFICIAL JOURNAL OF
THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India (AMTI) was started in 1965 for the promotion of efforts to improve Mathematics education at all levels. A major aim of the Association is to assist practising teachers of Mathematics in schools in improving their expertise and professional skills. Another important aim is to spot out and foster Mathematical talent in the young. The Association also seeks to disseminate new trends in Mathematics education among parents and public. Other activities of the Association include consultancy services to schools in equipping the Mathematics section of their libraries, in organising children's Mathematics clubs and fairs, in setting up teacher centres in schools, in conducting Mathematics laboratory programmes, in holding practical tests in Mathematics and in assisting children participating in investigational projects.

The Association holds "Inter – State Mathematical Talent Search Competition" annually and organises Orientation Courses, Seminars and Workshops for teachers and Summer Courses for talented students. A National Conference is held annually in different parts of the country for teachers to meet and deliberate on important issues of Mathematics education. Innovative teacher award has been instituted to give public recognition to enterprising and pioneering teachers of Mathematics for which entries from teachers are invited.

An award for contributions to the Mathematics Teacher relating to History of Mathematics in the context of mathematics education has been instituted by *Prof. R.C.Gupta.*

"The Mathematics Teacher (India)" (MT) is the official journal of the Association and is published quarterly. It has been approved for use in schools and colleges by the Government departments of education in many States. Besides MT the Association also bringsout Junior Mathematician (JM), a quarterly, specially for school students in three languages – English, Hindi and Tamil.

The membership of the Association is open to all those interested in Mathematics and Mathematics Education. The membership fee inclusive of subscription for the MT and effective from April 2001 is:

Subscription for India*

Category	Individual	Institutional
Life	Rs. 500	Rs. 1000
Annual (Ordinary)	Rs. 50	Rs. 100
Junior Mathematician – Life	Rs. 250	Rs. 20
Junior Mathematician – Annual	Rs. 25	Rs. 25

The Journal "The Mathematics Teacher" will be supplied free to all members.

* **For countries other than India same figures in US Dollar.**
i.e. instead of rupees read U.S. Dollars.

The Mathematics Teacher
(INDIA)

OFFICIAL JOURNAL OF THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Volume: 37 Issue: 3 & 4 Year: 2001

Prof. A.Ramakrishnan *Editor*

Prof. M.Srinivasan *Joint Editor*

EDITORIAL

In this combined third and fourth issues of Volume 37 of the Mathematics Teacher, besides contributions from mathematicians are included some class room notes and some card board constructions which can be advantageously used in the class room. The solutions to the problems of the screening and final tests of AMTI talent competitions will be published in the next issue. AMTI conducted a correspondence course on non-routine problem solving. The text prepared for this course has been published as a separate book edited by Prof. V.K.Krishnan, the details of which are published in this volume separately. Readers of MT and members of AMTI are requested to purchase this book and be profited by it. A report on the thirty sixth annual conference of AMTI held during 27-29, December, 2001 in Cochin also appears in this volume.The texts of special lectures delivered at the conference will be published in forthcoming issues.

EDITOR

CONTENTS

Association Activities

Prof T.R.Raghava Sastri Centenary Committee has agreed to publish reports of our workshops conducted in eight different parts of the country-Kancheepuram, Ranipet, Hanovar, Goa, Bhuvaneshwar, Surat, Delhi and Tinsukia, a few years back. They have earmarked a sum of Rs.1,00,000/- for this purpose of which Rs.25ggh,000/- has been handed over to us already. Sri P.K.Srinivasan, the eminent Mathematics education consultant, has kindly agreed to edit the same. We hope and trust that the published material will be available for use in the next academic year for teachers, libraries, training colleges and others interested in **innovative methods** of teaching-learning Mathematics. AMTI takes this opportunity to thank Prof. T. R. Ragavasastri centenary committee for their generous grant.

✱

Prof. T. Totadri Iyengar Centenary Committee has planned a project to orient high / higher secondary teachers in the Olympiad, IIT type problems solving. They plan to conduct the workshops in the places connected with the work and life of the late professor. The AMTI has been requested to lend the services of Resource Persons for which Prof. V. Shankaram has been nominated as coordinator.

Secretary, AMTI.

✱

PARABOLA SIMILARITY: ANALYSIS AND DISCUSSION

Bonnie H. Litwiller and **David R. Duncan**

Professors of Mathematics
University of Northern Iowa
Cedar Falls,Iowa 50614-0506

A central geometry concept is that of similarity. Students usually relate similarity to polygons. The idea that two triangles are congruent (the ratios of the sides are equal) is a familiar notion, suggesting that they have the same shape, but different sizes. It is more difficult to apply the similarity concept to "curves".

Many mathematics students have heard that all parabolas are similar, but they sometimes have trouble with this idea. For instance, the graphs of $y = 4x^2$ and $y = \frac{1}{4}x^2$ seem to have quite different shapes: (see Figure 1).

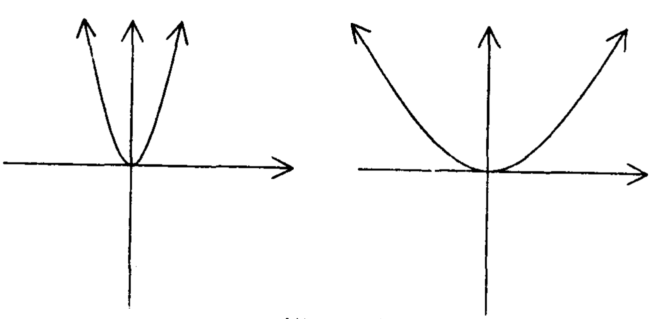


Figure 1

It is sometimes difficult to see how either of these parabolas could be enlarged or contracted to coincide with the other.

Similarity of polygons is easier to discuss since it involves line segments which can be measured and from which ratios can then be formed. What distances can be compared on parabolas? To analyze what is measurable on a parabola, we return to its derivation. A parabola is, by definition, the set of all points equidistant from a fixed point and a fixed line. Figure 2 depicts a fixed line d and a fixed point F. Also, a coordinate system (broken line) has been imposed so that the origin, 0 is equally distant from F and d. The distance from the origin to each of F and d is denoted as a.

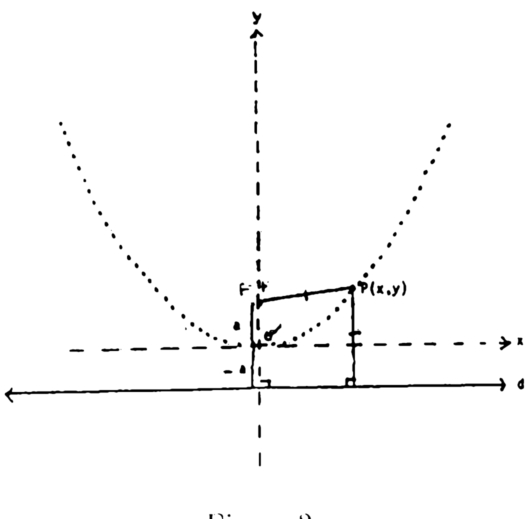


Figure 2

The points on the parabola are indicated as a series of dots. Let $P(x, y)$ be an arbitrary point on the parabola.

Then P is equally distant from F and d. Note that the coordinates of F are $(0, a)$ and the equation of the line d is $y = -a$.

The distance from P to F is $\sqrt{x^2 + (y-a)^2}$ and the distance from P to d is $y+a$. The two distances are equal, so:

$$\begin{aligned} y + a &= \sqrt{x^2 + (y-a)^2} \\ (y+a)^2 &= x^2 + (y-a)^2 \\ y^2 + 2ay + a^2 &= x^2 + y^2 - 2ay + a^2 \\ 4ay &= x^2 \end{aligned}$$

Figure 3 displays this parabola $x^2 = 4ay$. F is called the focus, d is called the directrix, and 0 is called the vertex of the parabola.

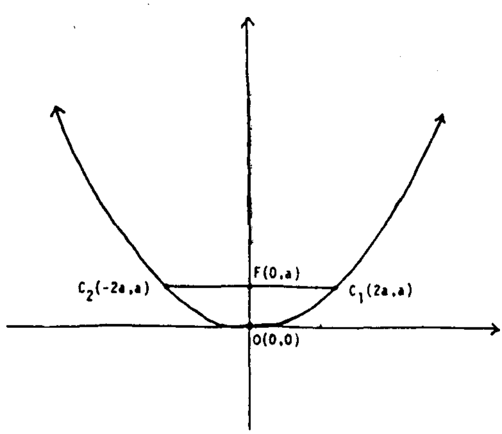


Figure 3

The line segment $\overline{C_1C_2}$ is called the focal chord. Note that each of the points C_1 and C_2 has y-coordinate a. Solving for x:

$$\begin{aligned} 4aa &= x^2 \\ x^2 &= 4a^2 \\ x &= \pm 2a \end{aligned}$$

C_1 has coordinates $(2a, a)$ and C_2 has coordinates $(-2a, a)$. The focal chord has length $4a$ while the semi-focal chord (F to C_1 or F to C_2) has length $2a$.

To establish similarity, consider two parabolas with equations $4ay = x^2$ and $4by = x^2$. Figure 4 displays these parabolas.

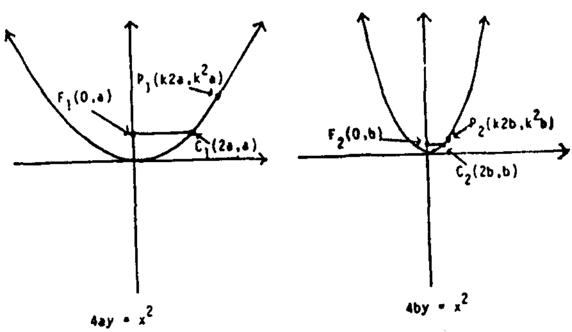


Figure 4

Let P_1 and P_2 have $x-$coordinates, respectively, of

$k(2a)$ and $k(2b)$; that is, the same multiples of the respective semi-focal chords. The $y-$ coordinate of P_1 is then found by solving:

$$\begin{aligned} 4ay &= [k(2a)]^2 \\ 4ay &= 4k^2a^2 \\ y &= k^2a. \end{aligned}$$

Similarity, the $y-$ coordinate of P_2 is k^2b. The ratio of the $x-$ coordinates of P_1 and P_2 is then $\dfrac{k \cdot 2a}{k \cdot 2b} = \dfrac{a}{b}$, and the ratio of the $y-$ coordinates of P_1 and P_2 is $\dfrac{k^2a}{k^2b} = \dfrac{a}{b}$.

These equal ratios are independent of the factor k. This indicates that the second parabola is a $\dfrac{b}{a}$ magnification (or contraction) of the first parabola.

Another indication of similarity can be found by computing the slopes of the tangents at P_1 and P_2. On parabola 1, $\dfrac{dy}{dx} = \dfrac{2x}{4a} = \dfrac{x}{2a}$; on parabola 2, $\dfrac{dy}{dx} = \dfrac{2x}{4b} = \dfrac{x}{2b}$. Replacing x with, respectively, $k(2a)$ and $k(2b)$ yields slopes of $\dfrac{k(2a)}{2a}$ and $\dfrac{k(2b)}{2b}$; both equal k.

The slopes of the corresponding points P_1 and P_2 are identical, thus indicating that the parabolas have identical shapes in their magnified or contracted forms.

CHALLENGE TO THE READER:

Does this same analysis hold for other conic sections?

CONSTRUCTIVISTIC IDEAS IN THE LEARNING OF MATHEMATICS

Frode Rønning
Faculty of Teacher Education
Sør-Trøndelag University College
N-7004 Trondheim, Norway

1. The Norwegian school system and teacher education

At the age of six children in Norway start a program of ten years of compulsory schooling. Theses ten years I will refer to as *primary school*, and I will split primary school into three levels: *low level* (grades 1 through 4), *mid level* (grades 5 through 7) and *high level* (grades 8 through 10). Almost all young people in Norway continue from primary school to secondary school which takes up to three years, depending on what line of study is chosen. There is a wide variety of lines to choose from in secondary school, from purely theoretical lines (preparing for university/college) to practical lines that train for specific professions.

There are, roughly speaking, two types of teacher education in Norway. The universities offer a teacher education program, which is based on the study of two or three subjects with a supplementary study of pedagogy and didactics, also including school practice. This program mainly educates teachers for secondary school and the high level of primary school. Most primary school teachers are

educated through a four year program which is offered by the university colleges. This program is an integrated program where subjects, pedagogy, didactics and school-practice are weaved together. Teachers with this education will have a broad, but not so deep, background in several subjects, but they have also certain possibilities of specializing in as much as the contents of the last year can be chosen (almost) freely. One year of study is set equal to 20 credit points, and in the first year all students have a 10 credit points course in mathematics in the fourth year so that the total can be 30 credit points. At the low and mid level the teacher teaches most of the subjects in his/her class, but this gradually changes, and at the high level it is common that each teacher only teaches a small number of subjects, according to his/her specialization.

2. The national curriculum

There exists a national curriculum both for the primary and secondary school, and the pedagogical ideas behind these curricula are to a large extent the same. I work with teacher education for primary school, and hence I am most familiar with the curriculum for primary school. I will therefore take my examples from this document. The current version of the national curriculum for primary school was presented in 1997. I will refer to this as L97 for short. A fundamental idea in L97 is that mathematics shall be a useful tool in various practical situations, and that it has important social and cultural aspects. This is emphasized by the fact that one of the main areas in the subject is called

Mathematics in daily life. The other main areas show more classical topics from the subject (see table below).

Level	Main area				
High level	Mathematics in daily life	Numbers and algebra	Geometry	Data-processing	Graphs and functions
Mid level	Mathematics in daily life	Numbers	Geometry	Data-processing	
Low level	Mathematics in daily life	Numbers	Space and shape		

3. The basic didactical principles.

The fact that 'Mathematics in daily life' is chosen to be one of the main areas (the first) reflects the attitude that mathematics can be found everywhere. Hence, in working with the subject we should look for applications in the life around us and not focus too much on what is in the textbooks. We should also be aware of how mathematics is used, sometimes unconsciously, in various activities. With this attitude it becomes natural to work with mathematics outdoors (doing measurements and calculations), combine mathematics and play, and include visits to art galleries and science centres as part of mathematics. Working in an interdisciplinary way becomes natural in this framework.

Mathematics has often been taught in a very deductive manner, with the teacher as the one who possesses the knowledge, and whose role it is to transfer this knowledge

to the students. In L97 the attitude towards learning mathematics (and also other subjects) is very much different from this view. The basic idea is that it is student him/herself who creates the knowledge, and that the teacher should assist the student in the process where knowledge is created. It is therefore fair to say that it is a constructivistic view on learning which is the basis for the work in Norwegian schools. Constructivism has to a large extent been developed by von Glaserfeld (1983) who identified the following consequences of a constructivistic view on the teaching process:

- Learning is separated from drill and practice
- The processes that are going on in the pupils' minds are more interesting than what shows in their exercises/problems
- Verbal communication is a process aiming at learning for one single pupil, not a process aiming at transferring knowledge

A natural consequence of this view is that the activities in the classroom shift from high degree of teacher activity to high degree of student activity. The student should actively take part in the learning process. Closely related to this view is the acceptance of the fact that the children possess valuable mathematical knowledge at the time they enter school. They use mathematics when they play, and in other situations in their everyday life. It is important that the school realises this competence, and that the children get

the understanding that their knowledge is valuable. The role of the school is to further develop the mathematical competence that is already there, and by working this way it is the hope that the children will feel that they themselves develop the knowledge and that they thereby are the owners of the knowledge. Mathematical knowledge should not be something, which is forced upon the students, but something, which grows from their own minds. Important are also the social relations between the students and the teacher, and among the students. Knowledge is constructed through a dialogue where the mathematical language is used actively, and through which the language is being developed and refined. This aspect of constructivism is often called *social constructivism* and is in particular developed by Berger and Luckmann (1966). In didactics of mathematics these ideas have been further developed by the Norwegian mathematics educator Stieg Mellin-Olsen (see e.g. Mellin-Olsen (1989)). A good example of an activity which is based on these ideas is described in Section 5 under the heading 'Geometric communication'.

The following quotation is taken from the national plan for mathematics in teacher education, and it presents some of the important guidelines in working with mathematics both in teacher education and in primary school.

> *The teaching of mathematics in primary school has for a long time been dominated by the learning of algorithms and the practising of fast and reliable computation. Today one has a more open view on the subject. The process of acquiring in-*

> *sight into mathematics is just as important as the algorithms themselves. Creativity and investigation is more important than before. Communication in and with mathematics, mathematics as a tool and a method in interdisciplinary work is very important.*

4. Working methods

A bearing principle in all subjects in Norwegian primary school is theme and project based work. It is not always easy to distinguish between theme work and project work, and the two concepts are often mentioned together. In L97 project work is defined as follows.

> *Project work is a way of working where the pupils, starting from a specific problem, problem area or a given task, define and carry out a piece of work from idea to finished product, concrete result or practical solution.*
>
> *Project work can be carried out both within one subject and as interdisciplinary work.*

Theme work has many of the same characteristic properties, but it is not as open ended as project work. It is more structured by the teacher, and therefore theme based work is more suitable for the young pupils, but as they grow older they will take more and more responsibility and the work tends to be more project oriented. L97 requires that at least 60% of the total time at the low level should be organized as theme work, at least 30% of the time at the mid level and at

least 20% of the time at the high level should be organized as theme and project work.

The Norwegian primary school is comprehensive in the sense that the pupils are at no stage sorted in different groups or classes according to their abilities. Even pupils with rather strong learning disabilities are placed in regular classes. This principle implies that almost every class has a broad spectrum of pupils as far as abilities in the various subjects are concerned, and therefore the need for individual tutoring is considerable, as well as the need for each and every pupil to advance at his/her own pace.

This situation, combined with constructivistic and inductive approach to the subject, has as a consequence that pure lecturing in mathematics is used more and more rarely. It may be used in an introductory and a conclusive phase, but the student does most of the work, alone or in groups. The students are supplied with a plan for a certain period of time (e.g. one week), which indicates what is supposed to be covered during this period in the various subjects. It is then to a large extent up to the students themselves to decide when and how they prefer to cover the indicated material. A large portion of the school day may be defined as 'working hours' where the students are free to work with whatever they like from the plan. The teacher(s) is (are) available for giving individual help. It should be said that not all schools or all teachers work in this way, but the ideas of L97 certainly encourage such flexible ways of working. A consequence of working in this manner is that the borders

between schoolwork and homework will disappear.

One particular way of working, which at the same time is both free and rather structured, should especially be mentioned. This is the so-called 'station teaching'. This means that the classroom, or the teaching area, is furnished with stations, each station containing certain exercises or activities to be carried out. Each station can take 4 to 6 pupils at a time, and during a given time (2-4 hours, say) each pupil in the group is supposed to have visited all the stations. At every station there should be activities at various levels of difficulty, and every pupil should choose what to do based on his/her ability. It could be that all the stations are focused on different aspects of the same topic, but this is not necessarily so.

This way of working demands much preparation by the teacher, but once the work has started it is very self-propelling, and the teacher can spend the time discussing with the pupils and give individual help. A model for station teaching, which is very fruitful is if one can accommodate two classes, with two teachers, in the same area. Then, if one of the teachers is busy discussing with a group of pupils, the other can overlook the area and be available if other pupils need help.

5. Some examples

Finally I will present some examples through which I will try to show how the constructivistic ideas from L97 can be carried out in practice.

Geometric communication

L97 emphasises communication with mathematical terms as an important goal. The following activity can be used at many levels, and it shows the importance of precise language and well-defined concepts.

Divide the class into pairs. Both persons in each pair make a drawing, using elements, which can be described with geometrical terms. An example of how such a drawing can be is shows in Figure 1, to the left.

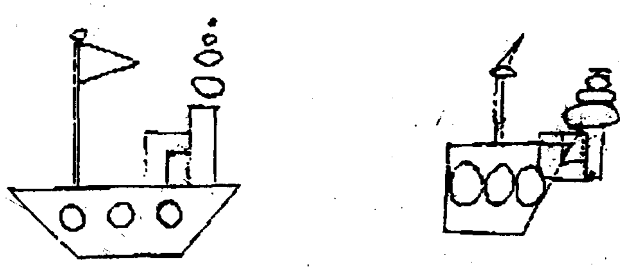


Figure 1

Now person A in the pair describes his/her drawing to person B, using only geometrical terms (rectangle, square, circle, midpoint) in addition to words like right, left, top, bottom. Person B tries to copy the drawing based on this description. The copy may look like the drawing above to the right. Now A and B compare the original and the copy and discuss why discrepancies occurred. What did or did not A say which made B draw something different from the original? This is a good exercise in using mathematical

language as well as a good exercise in expressing oneself in a precise way. Afterwards the roles change, and B describes his/her drawing to A who tries to copy it.

The land of 5 and 7 rupees

Imagine a situation where the only pieces of money are coins of 5 and 7 rupees. Based on this situation we can start to ask questions. Say that you want to buy something which cost 21 rupees, how can you pay? Choose other prices. What if the price is 2 rupees? Then the question of giving change comes up, and you can work with other prices where change is needed. This activity gives a good opportunity to practice multiplication. It also evokes certain questions: Can you pay any price with only 5 and 7 rupees? Why? What if the coins were changed to 4 and 6 rupees?

Instead of only doing multiplication as drill and practice we have the possibility to ask questions, do investigations, make conjectures and may be prove or disprove them.

Exploring the 2^{nd} degree curve

This example shows an inductive approach to the quadratic curve $y = ax^2 + bx + c$ using a computer program. The open question can be to try and find out as much as possible about how the values of a, b and c influence the shape of the curve.

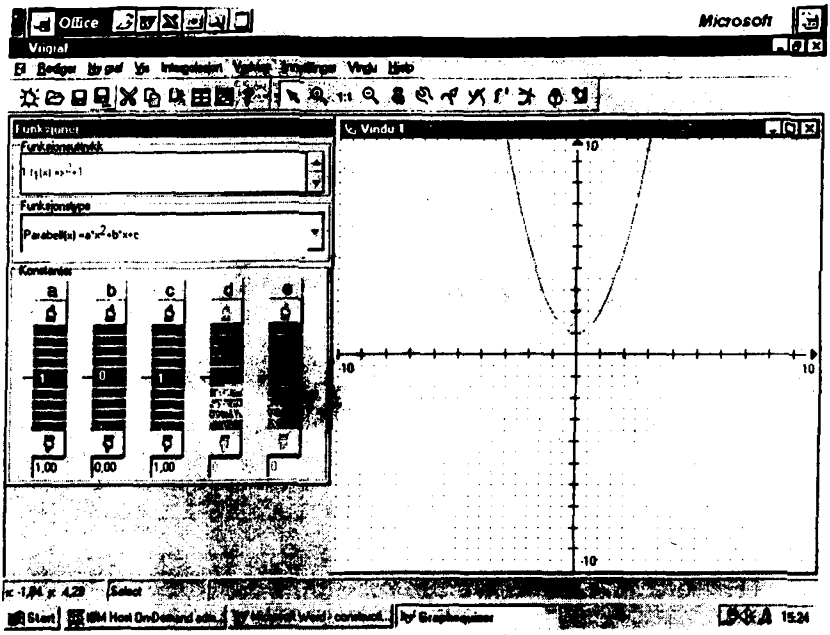


Figure 2

We see the curve $y = x^2 + 1$, and to the left in the picture we see that we can change the values of a, b and c. The effect is immediately shown on the graph. Working systematically by keeping two of the parameters fixed and varying the third we can generate conjectures about the role of the parameters, and afterwards we can test if the conjectures seem to be correct.

Visualising the conics by paper folding

In this example we shall see how the ellipse can be visualised by paper folding. From a piece of paper, cut a circle with radius about 20 cm. Mark a point F inside the circle, but not in the centre. Make a fold so that a point F' on the circumference gets mapped onto F (see Figure 3).

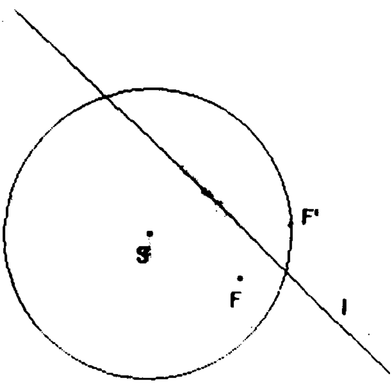


Figure 3

Let P be the point where the line SF' intersects the fold (see Figure 4). It is clear that $PF = PF'$, and therefore $SP + PF$ is a constant, namely the radius of the circle. Therefore, by varying F', the point P will describe an ellipse with S and F as foci. The fold will be a tangent to the ellipse. Making a large number of folds (same F, but changing F') we get a large number of tangents, and we can see the shape of the ellipse, better and better the more folds we make. This exercise is also well suited for a geometrical construction program on the computer. Such a program can draw the locus of the point P when F' varies, and we see the ellipse. This is how Figure 4 is made.

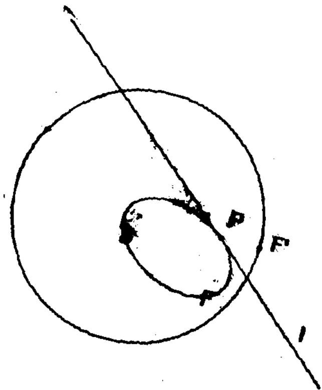


Figure 4

Cutting of the snow crystal

As a final example I will take an activity, which is particularly relevant for Norwegian children. Most children in Norway enjoy playing in the snow at wintertime, and there are rich possibilities for activities of various kinds in the school where snow is involved. Looking at the snow under a microscope can be fascinating, because the snow crystals have a beautiful structure. The pupils will soon discover that, although the crystals are different, they all have six-fold symmetry. Then we can start to discuss how we can make the shape of a snow crystal by cutting in a piece of paper. Most children have probably tried to fold a piece of paper several times, then cut along the edges, and fold out again. Usually this leads to four-fold or eight-fold symmetry, but now we want six-fold symmetry. How to do this?

Take a quadratic piece of paper, and fold it in two along one of the diagonals to form a triangle. Now fold from both sides so that we get an angle of 60 degrees pointing downwards (see Figure 5). Fold in two again. Cut away the top and cut along both edges in a pattern of your choice. Now fold out again, and we have a snow crystal!

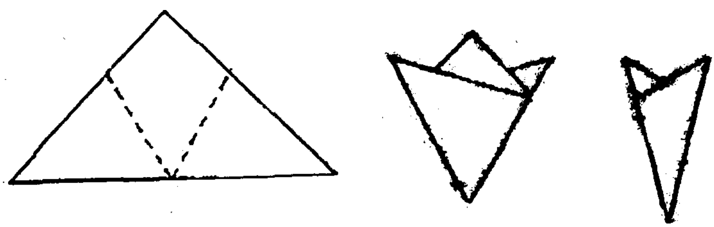


Figure 5

References:

1. **Berger P.** and **Luckmann T.** (1972), The Social Construction of Reality, Penguin Books, Hammondsworth.
2. L97 – Norwegian National Curriculum for Primary School, Ministry of Education, Research and Church Affairs (1996).
3. **Mellin-Olsen, S.** (1989), Kunnskapsformidling, Caspar Publishing Co., Bergen, Norway.
4. **von Glaserfeld, E.** (1983), Learning as a constructive activity. In *Proceedings from PME-NA, Montreal* 1983, 41-69.
5. **von Glaserfeld, E.** (1983), Learning as a constructive activity. In janvier, C. (ed.) *Problems of Representation in the Teaching and Learning of Mathematics.* Hillsdale: Lawrence Erlbaum, 3-17.

ON TANGENTS AND CONJUGATE DIAMETERS OF A HYPERBOLA

V. Balakrishnan
2mm 8, 5^{th} Cross.Street, Tagore Nagar,
Lawspet, Pondicherry - 605 008.

The paper consists of two parts. The first part deals with the tangents from a point to a hyperbola and the second is concerned about the conjugate diameters of a hyperbola.

1. On tangents from a point to a hyperbola

Consider the hyperbola $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$. This divides the plane into two regions, say R_1 and R_2, where R_1 contains the foci while R_2 does not contain the foci but contains the centre. Given a point $P(x_1, y_1)$ let S_{11} denote the number $S_{11} \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$. It is well known that $S_{11} > 0$ for all P in R_1 and $S_{11} < 0$, for all P in R_2. This paper proposes to investigate the location / orientation of the point $P(x_1, y_1)$ in respect of the following queries.

Query 1:

Where should be the point P be if the tangents from it to $S = 0$ are real and distinct?

Answer: P should be in R_2.

Proof. Let $Q(\alpha, \beta)$ be the point of contact of a tangent

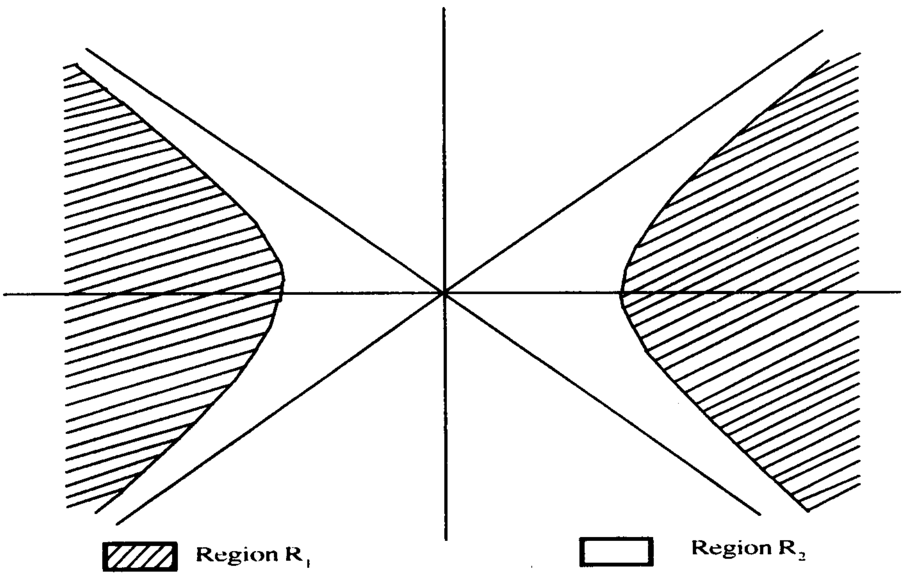


Figure 1:

from $P(x_1, y_1)$ to the hyperbola $S = 0$. Then we have

$$\frac{\alpha_1^2}{a^2} - \frac{\beta_1^2}{b^2} - 1 = 0 \qquad (1)$$

$$\frac{x_1\alpha_1}{a^2} - \frac{y_1\beta_1}{b^2} - 1 = 0 \qquad (2)$$

(since the tangent at Q passes through $P(x_1, y_1)$)

Eliminating β from (1) & (2), we get α as a root of the quadratic

$$x^2(\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}) - 2xx_1 + a^2(1 + \frac{y_1^2}{b^2}) = 0 \qquad (3)$$

Equation (3) gives the x-coordinates of the points of contact of the two tangents from P to $S = 0$. The two tangents will be real and distinct according as the two roots of (3) are real and distinct $\Rightarrow$ Discriminant of (3) $> 0 \Rightarrow \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 < 0$ whereby P should be in R_2.

Query 2:

Granting that $P(x_1, y_1)$ lies in R_2, where should P be located in R_2 so that both the tangents from P touch the same branch of $S = 0$?

To answer this we consider a further subdivision of the region R_2 by the asymptotes of $S = 0$. Let R_{21} be the subregion of R_2 between the curve $S = 0$ and the pair of asymptotes. Let $R_{22} = R_1 - R_{21}$.

Answer to Query 2: The two tangents from P will touch the same branch of $S = 0$ iff $P \in R_{21}$.

Proof. Suppose the two tangents PQ and PR from P to the hyperbola $S = 0$ touch the same branch at $Q \& R$. This means that the x coordinates of Q and R are either both positive or negative. i.e., the product of the x coordinates of $Q \& R$ is positive.

This means the product of the roots of equation (3) is positive $\Rightarrow \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 > 0 \Rightarrow \frac{y_1^2}{x_1^2} < \frac{b^2}{a^2} \Rightarrow$ the slope of OP should be between $\frac{-b}{a}, \frac{b}{a}$ which are the slopes of the asymptotes $\Rightarrow P \in R_{21}$.

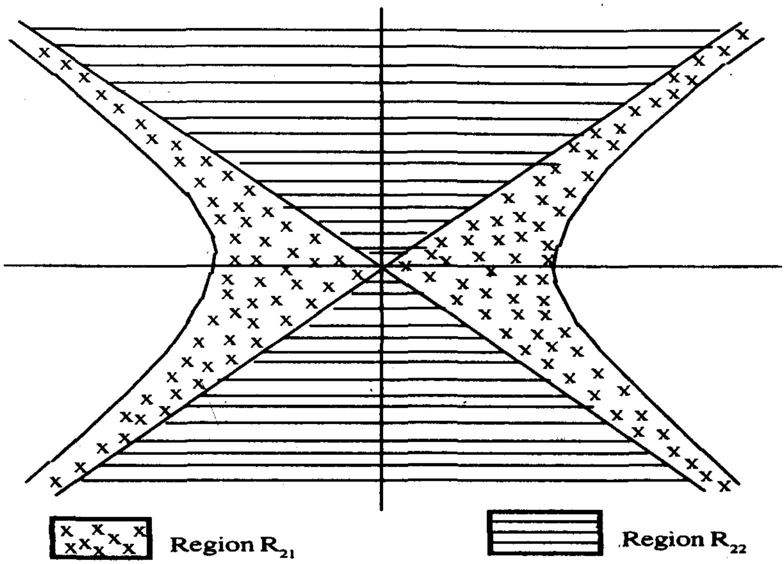


Figure 2:

Conclusion

If the two tangents from a point P to a hyperbola are real and distinct then P should be on the region of plane containing the conjugate axis and bounded by the hyperbola and vice versa.

Further both the tangents touch the same branch if and only if the point P lies in the region bounded by the asymptotes and the hyperbola.

If the point P is in the region of the plane bounded by the asymptotes and containing the conjugate axis, then

of the two tangents, one touches one branch and the other touches the second branch.

2. On conjugate diameters of a hyperbola

Consider the hyperbola $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$ and the conjugate hyperbola $S' \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$. The lines $y = \pm\frac{b}{a}x$ are asymptotes to both of them. The origin O is the centre for both $S = 0$ and $S' = 0$. Let the asymptotes divide the plane into two regions, say R_1 and R_2 where R_1 contains the curve $S = 0$ and R_2 contains $S' = 0$. It is also known that if $y = mx$ is a diameter for $S = 0$ then the conjugate diameter is $y = m'x$ where $mm' = \frac{b^2}{a^2}$. We start with the following.

Result 1:

If the diameter $y = mx$ cuts $S = 0$ at two real points say P' and P, then the conjugate diameter $y = m'x$ (where $mm' = \frac{b^2}{a^2}$) cuts the conjugate hyperbola $S' = 0$ at two real points say D' and D.

Proof. If $y = mx$ cuts $S = 0$ at $P'\&P$ (real points) then the slope m should be between $\frac{-b}{a}$, $\frac{b}{a}$ i.e., the slopes of the asymptotes i.e.,

$$\frac{-b}{a} < m < \frac{b}{a} \tag{1}$$

Since $mm' = \frac{b^2}{a^2}$, the slope m' of the conjugate diameter is either $< -\frac{b}{a}$ or $> \frac{b}{a}$. In other words if $y = mx$ lies in R_1, $y = m'x$ lies in R_2 and hence cuts $S' = 0$ at real

points D' and D. Hence the result.

Our next effort is to locate the point D (or D') on $S' = 0$ given the point P (or P') on $S = 0$. This leads to

Result 2:

If $P(a\ sec\theta, b\ tan\theta)$ is any point on $S = 0$ (with center O) then the diameter OD conjugate to OP cuts $S' = 0$ at $D(a\ tan\theta, b\ sec\theta)$ and $D'(-a\ tan\theta, -b\ sec\theta)$.

Proof. Slope of $OP = \frac{b\ tan\theta}{a\ sec\theta} = \frac{b}{a}\ sin\theta$

Therefore slope of $OD = \frac{b}{a\ sin\theta}$ (because $mm' = \frac{b^2}{a^2}$).

The points of intersection of OD with $S' = 0$ can be easily derived as $D(a\ tan\theta, b\ sec\theta)$ and $D'(-a\ tan\theta, -b\ sec\theta)$.

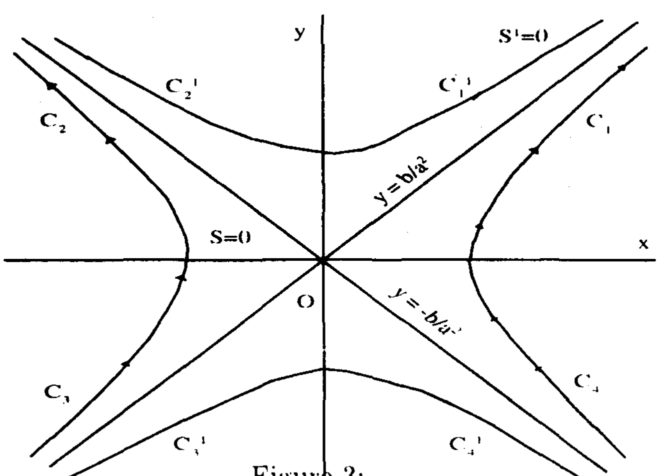


Figure 3:

One to one correspondence between $S = 0$ and $S' = 0$

Let C_i denote that part of the curve $S = 0$ in the i^{th} quadrant($i = 1$ to 4) of the plane as divided by the coordinate axes. Let θ vary in $(-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$.

For each θ we get a point $P(\theta)$ on the curve C i.e., $S = 0$ given by $(a \sec\theta, b\ tan\theta)$. As θ varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the point P starts from the infinite end of C_4 along the curve and moves towards the infinite end of C_1. (See the arrow mark in Figure 3), Similarly as θ varies in $(\frac{\pi}{2}, \frac{3\pi}{2})$, P starts from the infinite end of C_3 and moves along the curve to the infinite end of C_2.

We shall effect a one-one correspondence between $S = 0$ and $S' = 0$ as follows. For $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$ associate the point $P(a\ sec\theta, b\ tan\theta)$ of $S = 0$ with the point $D(a\ tan\theta, b\ sec\theta)$ of $S' = 0$. By this mapping $C_4 \to C_2^1$, $C_1 \to C_1^1$; $C_2 \to C_4^1$ and $C_3 \to C_3^1$. This correspondence associates with each point P on $S = 0$ the point D on $S' = 0$ which is the corresponding extremity of the diameter OD which is conjugate to OP.

Geometrical significance of the locations of P and D

Suppose l and m ate two straight lines and P and Q are points such that PQ is parallel to l but is bisected by m, then Q is called the reflection of P on m, reflection taken along l (see Figure 4).

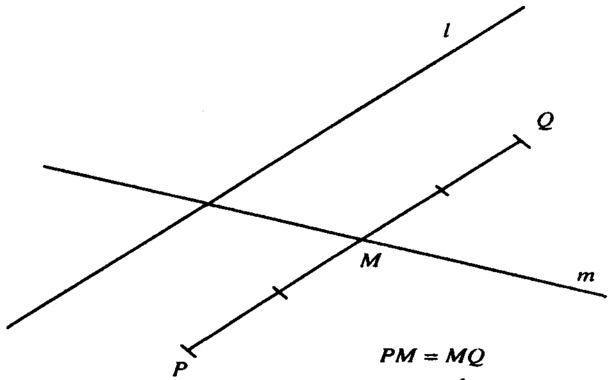


Figure 4:

It is now easy to see that the point $D(a\tan\theta, b\ sec\theta)$ is nothing but the reflection of $P(a\sec\theta, b\ tan\theta)$ on the asymptote $y = \frac{b}{a}x$, the reflection taken along the other asymptote $y = -\frac{b}{a}x$. For slope of $DP = \frac{b(sec\theta - tan\theta)}{a(tan\theta - sec\theta)} = -\frac{b}{a}$ so that DP is parallel to $y = -\frac{b}{a}x$. Midpoint of DP is $(\frac{a(tan\theta + sec\theta)}{2}, \frac{b(sec\theta + tan\theta)}{2})$ which lies on $y = \frac{b}{a}x$. Thus we get

Result 3:

For any point P on the hyperbola $S = 0$, the reflection of P on one asymptote, reflection taken parallel to the other asymptote gives the point D, which is an extremity of the diameter OD conjugate to OP.

A PAIR OF INTERSECTING LINES AND THE ASSOCIATED BASIC LINES

Dr. E. Rukmangadachari, M.A.,
M.Phil.,Ph.D.,M.N.A.Sc.,
Plot 12, H.No. 8-149/1, Prasoona Nagar,
Chintal, H.M.T. Road, Hyderabad - 500 054.

Balakrishnan's note [1] discusses the interesting problem: given the equation of a pair of intersecting lines, to plot geometrically the three associated basic lines. But the use of slopes in the solution restricts its general application. For instance, if any of the lines is a vertical line then its slope is not defined and if $m_1 + m_2 = 0$ then the Harmonic Mean of m_1 and m_2 is not defined. Balakrishnan and Sambasiva Rao [2] treat the same problem by a different approach. But in their proposition, which they call 'result' and in its proof they make the assumptions:

(i) $a \neq 0, b \neq 0$ (implicity)

(ii) the pair of lines are intersecting and the point of intersection is not on the $x-$ axis or the $y-$ axis

though later on they discuss some particular cases.

The interest aroused by these notes has resulted in thinking of a different analytical approach to the problem, which takes care of all the cases and it is presented here. The following results could be easily proved or could be found

in any standard Intermediate (or P.U.C. or plus two level) Mathematics Text Book [3],[4].

(1). $$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \qquad (1)$$

represents a pair of straight lines $\Longleftrightarrow$

$$h^2 \geq ab, g^2 \geq ac, f^2 \geq bc, \Delta \equiv abc+2fgh-af^2-bg^2-ch^2 = 0 \qquad (2)$$

(2). If $S = 0$ represents a pair of lines then

$$S_0 \equiv ax^2 + 2hxy + by^2 = 0 \qquad (3)$$

represents a pair of lines parallel to the pair $S = 0$ and passing through the origin.

The pair of lines are intersecting if $h^2 > ab$ and are parallel (or coincide) if $h^2 = ab$.

(3). If $S = 0$ represents the pair of lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$ intersecting at C (different from the origin O) then C is the point of concurrence of the following three basic lines:

$$ax + hy + g = 0 \qquad (\alpha)$$

$$hx + by + f = 0 \qquad (\beta)$$

$$gx + fy + c = 0 \qquad (\gamma)$$

(4). If $S_0 = 0$ represents the pair of lines $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$ through the origin parallel to the lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$ respectively given by $S = 0$ then the four lines form a parallelogram $OACB$ whose diagonal $\overleftrightarrow{AB}$ is given by

$$S - S_o \equiv 2gx + 2fy + c = 0$$

We now take up the problem of plotting the basic lines $(\alpha), (\beta)$ and (γ).

Case (A) Intersecting Lines: $h^2 > ab$

First we consider the case when the lines $S = 0$ are intersecting at C (different from the origin O) i.e. when $h^2 > ab$. From the result (4) above we observe that the basic line (γ) is the line passing through C and parallel to the diagonal $\overleftrightarrow{AB}$ and so it can be plotted.

We now take up equation (1) and consider the issue of plotting the lines (α) and (β). Two cases arise:

(i) $b \neq 0$ and

(ii) $b = 0, h \neq 0 (b = 0$ and$h = 0$ is impossible since $h^2 > ab)$

Subcase (i). $b \neq 0$, Since $h^2 > ab$ and $\Delta = 0$ we can write S as follows:

$$S = \tfrac{1}{b} \times [(by + hx + \sqrt{(h^2 - ab)} + \lambda)] \times [(by + hx - \sqrt{(h^2 - ab)} + \mu)] \qquad (5)$$

where the equations

$$\left.\begin{aligned} \lambda + \mu &= 2f \\ h(\lambda + \mu) - \sqrt{(h^2 - ab)(\lambda - \mu)} &= 2bg \\ \Rightarrow (\lambda - \mu) &= \frac{(2fh - 2bg)}{\sqrt{h^2 - ab}} \\ \lambda \times \mu &= b \times c \\ &= \frac{f + (fh - bg)}{\sqrt{(h^2 - ab)}} \\ &\times \frac{f - (fh - bg)}{\sqrt{(h^2 - ab)}} \end{aligned}\right] \qquad (6)$$

are consistent yielding the values of λ and μ :

$$\lambda = [f + (fh - bg)/\sqrt{(h^2 - ab)}]$$

$$\mu = [f - (fh - bg)/\sqrt{(h^2 - ab)}]$$

So, we have the equations of $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$; and those of $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$ as follows:

$$\begin{aligned} \overleftrightarrow{BC}\colon L &\equiv by + hx + \sqrt{(h^2 - ab)}x + f \\ &\quad +(fh - bg)/\sqrt{(h^2 - ab)} = 0 && (7) \\ \overleftrightarrow{AC}\colon L' &\equiv by + hx - \sqrt{(h^2 - ab)}x \\ &\quad +f - (fh - bg)/\sqrt{(h^2 - ab)} = 0 && (8) \\ \overleftrightarrow{OA}\colon L_o &\equiv by + hx + \sqrt{(h^2 - ab)}x = 0 && (9) \\ \overleftrightarrow{BC}\colon L'_o &\equiv by + hx - \sqrt{(h^2 - ab)}x = 0 && (10) \end{aligned}$$

The straight lines $y = a$ and $x = b$ intersect the lines (9) and (10) at

$$\left.\begin{aligned} D &= (-h + \sqrt{h^2 - ab}, a) \\ E &= (-h - \sqrt{h^2 - ab}, a) \end{aligned}\right] \qquad (11)$$

$$\left.\begin{aligned} D' &= (b, -h - \sqrt{h^2 - ab}\ ) \\ E' &= (b, -h + \sqrt{h^2 - ab}\ ) \end{aligned}\right] \qquad (12)$$

The midpoints of $\overleftrightarrow{DE}$ and $\overleftrightarrow{D'E'}$ are $F = (-h, a)$ and $F' = (b, -h)$ respectively. So, the equations of $\overleftrightarrow{OF}$ and

$\overleftrightarrow{OF'}$ are:

$$\overleftrightarrow{OF}\colon ax + hy = 0 \tag{13}$$

$$\overleftrightarrow{OF'}\colon hx + by = 0 \tag{14}$$

We therefore observe that the basic lines $(\alpha), (\beta)$ and (γ) are the lines $\overleftrightarrow{CP}, \overleftrightarrow{CQ}$ and $\overleftrightarrow{CR}$ passing through C and parallel to $\overleftrightarrow{OF}, \overleftrightarrow{OF'}$ and $\overleftrightarrow{AB}$ respectively (Fig.1-3,7-9).

Subcase (ii). $b = 0, h \neq 0$

When $b = 0$ we have $h \neq 0$ since $h^2 > ab$ and $\Delta = 0 \Rightarrow 2fgh - af^2 - ch^2 = 0$. So, S can be written as

$$\begin{aligned} S &\equiv 2hxy + ax^2 + 2gx + 2fy + c \\ &\equiv (ax + 2hy + 2g + \lambda) \times (x + \mu) \end{aligned} \tag{15}$$

where

$$\left.\begin{array}{rcl} 2g + \lambda + \mu a = 2g & & \lambda = -fa/h \\ \text{and} & \Rightarrow & \text{and} \\ 2h\mu = 2f & & \mu = f/h \\ (2g + \lambda)\mu = c & \Rightarrow & (2g - fa/h)(f/h) = c \\ & \Rightarrow & 2fgh - af^2 - ch^2 = 0 \end{array}\right] \tag{16}$$

are consistent; so that we have the equations of $\overleftrightarrow{BC}, \overleftrightarrow{AC}, \overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$ as follows:

$$\overleftrightarrow{BC} \;:\; L \equiv ax + 2hy + 2g - fa/h = 0 \tag{17}$$

$$\overleftrightarrow{AC} \;:\; L' \equiv x + f/h = 0 \tag{18}$$

$$\overleftrightarrow{OA} \;:\; L_0 \equiv ax + 2hy = 0 \tag{19}$$

$$\overleftrightarrow{OB} \;:\; L'_0 \equiv x = 0 \tag{20}$$

The straight line $y = a$ intersects $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$ at $D = (-2h, a)$ and $E = (0, a)$

The mid-point of $\overleftrightarrow{DE}$ is $F = (-h, a)$ and so, the equation of $\overleftrightarrow{OF}$ is

$$ax + hy = 0 \tag{21}$$

Also,when $b = 0$ equation (β) becomes

$$hx + f = 0 \tag{22}$$

In this case,we observe that the basic lines $\overleftrightarrow{CP}, \overleftrightarrow{CQ}$ and $\overleftrightarrow{CR}$ through C are parallel to $\overleftrightarrow{OF}, \overleftrightarrow{OB}$ and $\overleftrightarrow{AB}$ respectively (Figs.4-6)

There arise the following nine cases corresponding to which nine figures(Figs.1-9) have been drawn.

1.$a > 0,\ b > 0$	4.$a > 0,\ b > 0$	7.$a > 0,\ b < 0$
2.$a = 0,\ b > 0$	5.$a = 0,\ b = 0$	8.$a = 0,\ b < 0$
3.$a < 0,\ b > 0$	6.$a < 0,\ b = 0$	9.$a < 0,\ b < 0$

Figures 1-3,7-9 correspond to the subcase(i) $b \neq 0$, and Figs.4-6 correspond to the subcase(ii) $b = 0,\ h \neq 0$.

Particular cases.

(i) $a = 0, b \neq 0$: L is parallel to the $X-$ axis and L' is not parallel to the X or $Y-$ axis.

Let $L \equiv y + \lambda = 0$ and $L' \equiv by + 2hx + \mu = 0$ so that their combined equation is

$$S \equiv LL' \equiv by^2 + 2hxy + (b\lambda + \mu)y + 2\lambda hx + \lambda\mu = 0 \tag{23}$$

Comparing this with equation (1) we have,

$$a = 0, \lambda = g/h, b\lambda + \mu = 2f, \lambda\mu = c \tag{24}$$

which are consistent since

$$\Delta = 0 \Rightarrow \lambda\mu = (2fh - bg)\frac{g}{h^2} = c \tag{25}$$

Therefore

$$L \equiv y + g/h = 0, L' \equiv by + 2hx + (2fh - bg)/h = 0 \tag{26}$$

In this case, when $b \neq 0, (\alpha)$ coincides with $L = 0$. The three basic lines are as illustrated in Figs.2 and 8 for the cases $b > 0$ and $b < 0$ respectively.

(ii) $a = 0, b = 0$ L is parallel to the $X-$ axis and L' is parallel to the $Y-$ axis

In this case,the equations of the given lines are:

$$L \equiv y + g/h = 0, L' \equiv hx + f = 0 \tag{27}$$

and (α) and (β) are coincident with these lines while (γ) coincides with the diagonal of the rectangle formed by $L = 0, L' = 0$ and the coordinate axes as depicted in Fig.5.

(iii) The point of intersection C lies on the $X-$ axis

If the point of the intersection of the given lines $C(x_1, y_1)$ lies on the $X-$ axis, then $y_1 = 0$ and its x coordinate x_1 satisfies $(\alpha), (\beta)$ and (γ) so that we have $ax_1 + g = 0, hx_1 + f = 0, gx_1 + c = 0$ implying

$$\frac{g}{a} = \frac{f}{h} = \frac{c}{g} \tag{28}$$

In this case (α) and (γ) are coincident and parallel to the diagonal $\overleftrightarrow{AB}$ of the parallelogram $OABC$ and (β) is parallel to $\overleftrightarrow{OF'}$ where $F' = (b, -h)$ as depicted in Fig.10.

(iv) The point of intersection C lies on the $Y-$ axis

In this case, $x_1 = 0$ and y_1 satisfies $(\alpha), (\beta)$ and (γ) so that $hy_1 + g = 0, by_1 + f = 0, fy_1 + c = 0$ which imply

$$g/h = f/b = c/f \tag{29}$$

In this case, (β) and (γ) are coincident and parallel to the diagonal $\overleftrightarrow{AB}$ of the parallelogram $OABC$ and (α) is parallel to $(\overleftrightarrow{OF})$ where $F = (-h, a)$ as shown in Fig.11.

Case(B) Parallel lines $h^2 = ab$.

We now consider the case when $h^2 = ab$. Without loss of generality, we can take a and b to be non-negative. Three cases arise:

(i) $a > 0, b > 0$

(ii) $a = 0, b > 0$

(iii) $a > 0, b = 0$

Subcase(i). $a > 0, b > 0$

Since $a/h = h/b$, the basic lines (α) and (β) are parallel. Also, $\Delta = 0$ implies

$$(\sqrt{a}f - \sqrt{b}g)^2 = 0 \Rightarrow g/\sqrt{a} = f/\sqrt{b} = k(\text{say}) \tag{30}$$

So,the equations (α) and (β) reduce to the single equation:

$$\sqrt{a}x + \sqrt{b}y + k = 0 \tag{31}$$

Further, $S = 0$ represents a pair of lines implies

$$g^2 \geq ac, f^2 \geq bc \Rightarrow g^2 + f^2 \geq c(a+b) \Rightarrow k^2(a+b)$$

$$\geq c(a+b) \Rightarrow k^2 \geq c \text{ (since } a+b>0) \tag{32}$$

Now, $S \equiv (\sqrt{a}x + \sqrt{b}y + k + \sqrt{(k^2-c)})(\sqrt{a}x + \sqrt{b}y + k - \sqrt{(k^2-c)})$ Thus, $S = 0$ represents the two parallel lines:

$$L \equiv \sqrt{a}x + \sqrt{b}y + k + \sqrt{(k^2-c)} = 0 \tag{33}$$

$$L' \equiv \sqrt{a}x + \sqrt{b}y + k - \sqrt{(k^2-c)} = 0 \tag{34}$$

which become coincident if $k^2 = c$.

In this case, the basic lines (α) and (β) are coincident, and are parallel to the above pair and lie midway between them. The basic line (γ) whose equation becomes $\sqrt{a}x + \sqrt{b}y + c/k = 0$ is also parallel to the given pair of the lines and can be plotted.

Subcase(ii). $a = 0, b > 0$ $\quad h^2 = ab \Rightarrow h = 0$ and $\Delta = 0 \Rightarrow g = k\sqrt{a} = 0$

Thus $a = h = g = 0$ and

$$\begin{aligned} S \equiv by^2 + 2fy + c &= (1/b)(by + f + \sqrt{(f^2-bc)}) \\ &\quad \times (by + f - \sqrt{(f^2-bc)}) \\ &= 0 (\text{since } f^2 \geq bc) \end{aligned} \tag{35}$$

So, $\quad S = 0$ represents the lines

$$L \equiv by + f + \sqrt{(f^2 - bc)} = 0 \tag{36}$$

$$L' \equiv by + f - \sqrt{(f^2 - bc)} = 0 \tag{37}$$

which are both parallel to $X-$ axis. The basic line (α) vanishes and the line (β) with its equation

$$by + f = 0$$

is parallel to these lines and lies midway between them. In this case (γ) becomes

$$fy + c = 0$$

so that it is parallel to the $X-$ axis along with the given lines and it can be plotted.

Subcase (iii). $a > 0, b = 0$ $\quad$ Since $h = b = f = 0$, S can be written as

$$\begin{aligned} S &\equiv ax^2 + 2gx + c \\ &= (1/a)(ax + g + \sqrt{g^2 - ac})(ax + g - \sqrt{g^2 - ac}) \\ &\quad \text{(since } g^2 \geq ac) \end{aligned}$$

So, $S = 0$ represents the lines

$$L \equiv ax + g + \sqrt{(g^2 - ac)} = 0 \tag{38}$$

$$L' \equiv ax + g - \sqrt{(g^2 - ac)} = 0 \tag{39}$$

which are parallel to the $Y-$ axis.

figures

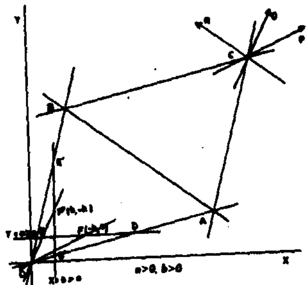


Figure. 1

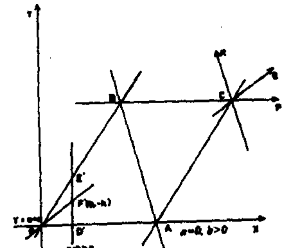


Figure. 2

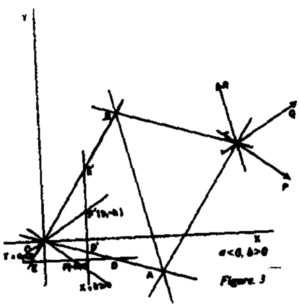


Figure. 3

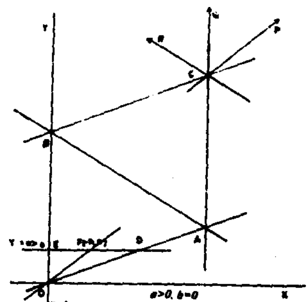


Figure. 4

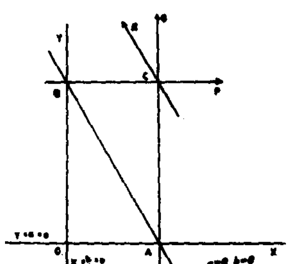


Figure. 5

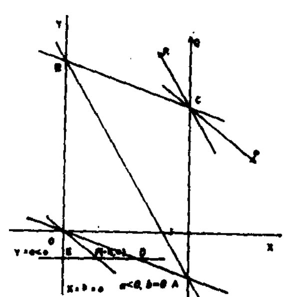
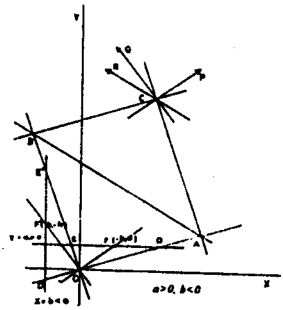


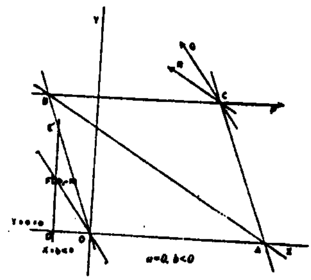
Figure. 6

figures



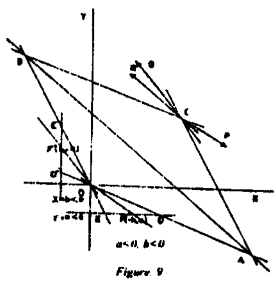
$a>0,\ b<0$

Figure. 7



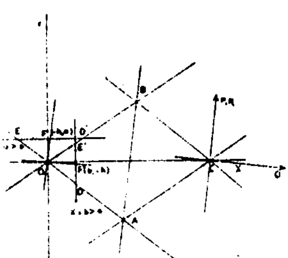
$a=0,\ b<0$

Figure. 8



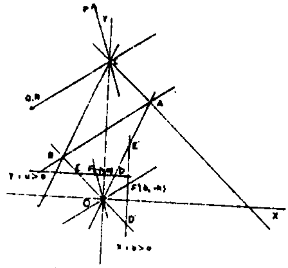
$a<0,\ b<0$

Figure. 9



(Intersection Point lies on the X-axis)
[illegible]

Figure. 10



(Intersection Point lies on the X-axis)
$a>0,\ b>0$

Figure. 11

The basic line (β) vanishes and the line (α) with its equation

$$ax + g = 0$$

is parallel to these lines and lies midway between them. In this case (γ) becomes

$$gx + c = 0$$

so that it is parallel to the $Y-$ axis along with the given lines and it can be plotted.

Remark. From analytical stand-point, the case of intersecting lines $(h^2 > ab)$ and that of parallel lines $(h^2 = ab)$ are different and from geometrical stand-point we may perhaps consider the latter as a 'limiting case' or as a 'case of degeneration' of the former one but not as a 'particular case'.

References:

1. **V.Balakrishnan** "Where are the three basic lines?" Math. Teacher Vol.35, No.3(1999) pp.172-174.
2. **V.Balakrishnan** and **N.Sambasiva Rao** "Where are the three basic lines? An alternative description" Math. Teacher Vol.36, Nos.1,2(2000) pp.46-53.
3. **V.Venkateswara Rao**, **N.Krishnamurthy** and **B.V.S.S.Sarma** Intermediate Mathematics Vol.1(1999) pp.535-537.
4. **M.Peri Sastry**, **SV Raghavendra Rao** et al: Junior Intermediate Mathematics(1987), Vikram Book Links, pp 119-135.

JARASANDHA NUMBERS[1]

P.S.N. Sastry
59 A-7-29, S.B.H. Colony, Plot No.3
Near Saibaba Temple, Vijayawada – 520008.

In our Indian Epic Mahabharatha, we come across a demonaic figure named Jarasandha. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body.

In the field of mathematics, we have numbers exhibiting the same property as Jarasandha. Consider a number of the form XC. This may be split as two numbers X and C, and if these numbers are added and squared, we get the same number XC again.

$$\text{i.e. } XC \to (X + C)^2 = 10^n X + C = XC$$

For example, examine the number $81 (X = 8, C = 1)$.

$$81 \to (8 + 1)^2 = (10^1 \times 8) + 1 = 81.$$

The number C should be n digits long. (n being the power to which 10 is raised before it is multiplied with X.) If not, zeroes may be added suitably as in the following example:

$$9801 = (98 + 01)^2 = 99^2 = 9801 = 10^2 \times 98 + 01 = 9801.$$

Let us solve the equation $(X + C)^2 = 10^n X + C$ for X :

$$X^2 + 2CX + C^2 = 10^n X + C$$

---

[1] Paper presented at the 35^{th} annual conference of AMTI held at Pune in December 2000.

or

$X^2+(2C-10^n)X+(C^2-C)=0$ a quadratic expression in X.

If $C = 1$, the equation becomes $X^2 + (2 - 10^n)X = 0$, which can be solved for different values of $n \neq 0$. (If $n = 0$, then C, which is n digits long will become non-existent.)

For $n = 1, X^2 + (2 - 10)X = 0$ or $X^2 - 8X = 0$, i.e., $X(X - 8) = 0$; therefore $X = 8$ (since $X \neq 0$) which means that $X = 8$ when $n = 1$ and $C = 1$. Therefore XC, i.e., 81 becomes a Jarasandha number which can be easily verified (example given above). Similarly,

For $n = 2, X = 98$. Therefore, $XC = 9801(C = 1$ suitably altered as 01, since $n = 2)$

$$\begin{array}{rllll}
81 & = (8+1)^2 & = 9^2 & = 81 \\
9801 & = (98+01)^2 & = 99^2 & = 9801 \\
998001 & = (998+001)^2 & = 999^2 & = 998001 \\
99980001 & = (9998+0001)^2 & = 9999^2 & = 99980001
\end{array}$$

and so on $\cdots$

And, if we fix the value of n as 2, for $9 < C < 100$ and under the condition that $(2500 - 99C)$ is a perfect square, we obtain two values for X.

Say, $C = 25$ for example: $(X + 25)^2 = 10^2X + 25$ or $X^2 - 50X + 600 = 0$.

Simplification gives two values for X, i.e., 30&20. Appending the value of C to these X, we get the two

Jarasandha numbers as 3025 and 2025.

$$\begin{aligned} 3025 &= (30+25)^2 &= 55^2 &= 3025 \\ 2025 &= (20+25)^2 &= 45^2 &= 2025 \end{aligned}$$

The following, and many more such numbers fit in the above pattern:

$$\begin{array}{rclclcl} 2025 &=& (20+25)^2 &=& 45^2 &=& 2025 \\ 3025 &=& (30+25)^2 &=& 55^2 &=& 3025 \\ 88209 &=& (88+209)^2 &=& 297^2 &=& 88209 \\ 494209 &=& (494+209)^2 &=& 703^2 &=& 494209 \\ 4941729 &=& (494+1729)^2 &=& 2223^2 &=& 4941729 \\ 7441984 &=& (744+1984)^2 &=& 2728^2 &=& 7441984 \\ 24502500 &=& (2450+2500)^2 &=& 4950^2 &=& 24502500 \\ 25502500 &=& (2550+2500)^2 &=& 5050^2 &=& 25502500 \\ 52881984 &=& (5288+1984)^2 &=& 7272^2 &=& 52881984 \\ 60481729 &=& (6048+1729)^2 &=& 7777^2 &=& 60481729 \end{array}$$

This can be generalized in the following way:

For $n = 2$ and for all Cs that are two digits long, if a pair of integer values for X can be obtained, satisfying the equation $X = 50 - C \pm \sqrt{2500 - 99C}$, such numbers will show the Jarasandha property.

Similarly, for $n = 3$ and for all Cs that are three digits long, the integer values of X fitting the equation $X = 500 - C \pm \sqrt{250000 - 999C}$ will exhibit this property.

Another interesting fact that emerges when we observe the $(X+C)$ values in the above set of numbers; we find pairs such as $(45, 55)$, $(297, 703)$, $(2223, 7777)$, $(2728, 7272)$, $(4950, 5050)$ etc. These number-pairs confirm that

- One of the two numbers is invariably a multiple of 11;
- The other is its 10^n complement, and
- These two numbers are equi-distant mirror reflections from $(10^n \div 2)$

Thus we can infer that, if any multiple of 11 exhibits the Jarasandha property, its 10^n complement will also do the same.

It also follows that the two values of X, say X_1 and X_2, derived by solving the quadratic equation always follow the rule $X_1X_2 = C^2 - C$.

This can be verified by substituting $(50 - C + \sqrt{2500 - 99C})$ for X_1 and $(50 - C - \sqrt{2500 - 99C})$ for X_2. (eg. From the Jarasandha pair (2025,3035), we can see that $20 \times 30 = 25^2 - 25 = 600$).

In the above numbers two numbers are paired together such that

45	&	55
297	&	703
2223	&	7777
2728	&	7272
4950	&	5050

In the above pairs one of the numbers in each pair is a multiple of '11', they are $55, 297, 7777, 2728, 4950 \cdots$

If we add each pair

45	+	55	=	100	2 digit number 10^2
297	+	703	=	1000	3 digit number 10^3
2223	+	7777	=	10000	4 digit number 10^4
2728	+	7272	=	10000	4 digit number 10^4
4950	+	5050	=	10000	4 digit number 10^4

Therefore when we add each pair sum of the numbers will be 10^n, where n is the number of digits in each number in the pair.

Similarly

1	+	9	=	10	1 digit number 10^1
1	+	99	=	100	2 digit number 10^2
1	+	999	=	1000	3 digit number 10^3
1	+	9999	=	10000	4 digit number 10^4

In the above pairs one number is '1' & the other numbers are $10^n - 1$ i.e., $9, 99, 999, 9999, \cdots, etc.$

On successive addition of the digits in the above numbers give either '9' or '10'

$$
\begin{aligned}
45 &= 4 + 5 &&= 9 \\
55 &= 5 + 5 &&= 10 \\
297 &= 2 + 9 + 7 &&= 18 = 1 + 8 = 9 \\
703 &= 7 + 0 + 3 &&= 10 \\
2223 &= 2 + 2 + 2 + 3 &&= 9 \\
7777 &= 7 + 7 + 7 + 7 &&= 10
\end{aligned}
$$

$$
\begin{aligned}
2728 &= 2+7+2+8 &= 10\\
7272 &= 7+2+7+2 &= 9\\
4950 &= 4+9+5+0 &= 9\\
5050 &= 5+0+5+0 &= 10
\end{aligned}
$$

$5 \pm 4 = 1\&9$	number of 1 digit $[\frac{(10)^1}{2} \pm$ (complement of 1&9 to 5)]
$50 \pm 5 = 45\&55$	number of 2 digits $[\frac{(10)^2}{2} \pm$ (complement of 45&55 to 50)]
$50 \pm 49 = 01\&99$	number of 2 digits $[\frac{(10)^2}{2} \pm$ (complement of 1&99 to 50)]
$500 \pm 203 = 297\&703$	number of 3 digits $[\frac{(10)^3}{2} \pm$ (complement of 297&703 to 500)]
$500 \pm 499 = 001\&999$	number of 3 digits $[\frac{(10)^3}{2} \pm$ (complement of 001&999 to 500)]
$5000 \pm 2777 = 2223\&7777$	number of 4 digits $[\frac{(10)^4}{2} \pm$ (complement of 2223&7777 to 5000)]
$5000 \pm 2772 = 2728\&7272$	number of 4 digits $[\frac{(10)^4}{2} \pm$ (complement of 2728&7272 to 5000)]
$5000 \pm 50 = 4950\&5050$	number of 4 digits $[\frac{(10)^4}{2} \pm$ (complement of 4950&5050 to 5000)]
$5000 \pm 4999 = 0001\&9999$	number of 4 digits $[\frac{(10)^4}{2} \pm$ (complement of 1&9999 to 5000)]

In the above pairs each number is equidistant to $\frac{10^n}{2} \pm$ com-

plements of $(n + c)$ to $\frac{10^n}{2}$

$$\begin{array}{lcll}
5 - 4 & = & 1 & 5 + 4 = 9 \\
50 - 5 & = & 45 & 50 + 5 = 55 \\
50 - 49 & = & 01 & 50 + 49 = 99 \\
500 - 203 & = & 297 & 500 + 203 = 703 \\
500 - 499 & = & 001 & 500 + 499 = 999 \\
5000 - 2777 & = & 2223 & 5000 + 2777 = 7777 \\
5000 - 2272 & = & 2728 & 5000 + 2728 = 7272 \\
5000 - 50 & = & 4950 & 5000 + 50 = 5050 \\
5000 - 4999 & = & 0001 & 5000 + 4999 = 9999
\end{array}$$

By this we can observe in each pair one number is the reflection of the other as they are equidistant to $\frac{10^n}{2}$ on either side.

It also follows that the two values of X, say X_1 and X_2, derived by solving the quadratic equations-always follow the rule $X_1X_2 = C^2 - C$.

$$\begin{aligned}
(20 + 25) \text{ and } (30 + 25)20 \times 30 &= 25^2 - 25 \\
&= 25 \times 24 \\
&= 600 \\
(88 + 209) \text{ and } (494 + 209)88 \times 494 &= 209^2 - 209 \\
&= 209 \times 208 \\
&= 43472
\end{aligned}$$

$$(494 + 1729) \text{ and } (6048 + 1729)$$

$$\begin{aligned}
494 \times 6048 &= 1729^2 - 1729 \\
&= 1729 \times 1728 \\
&= 2987712
\end{aligned}$$

(744 + 1984) and (5288 + 1984)

$$744 \times 5288 = 1984^2 - 1984$$
$$= 1984 \times 1983$$
$$= 3934272$$

(2450 + 2500) and (2550 + 2500)

$$2450 \times 2550 = 25002^2 - 2500$$
$$= 2500 \times 2499$$
$$= 6247500$$

When we multiply the squares of the numbers in each pair we get $[(100^n - 1)C]^2$ where n is the number of digits of the number.

$45^2 \times 55^2$	$= [99 \times 25]^2$	$= [(10^2 - 1) \times 25]^2$
$297^2 \times 703^2$	$= [999 \times 209]^2$	$= [(10^3 - 1) \times 209]^2$
$2223^2 \times 7777^2$	$= [9999 \times 1729]^2$	$= [(100^4 - 1) \times 1729]^2$
$2728^2 \times 7272^2$	$= [9999 \times 1984]^2$	$= [(100^4 - 1) \times 2500]^2$

Similarly there are some numbers which, when split, one part subtracted from the other and the result squared, gives back the original number. (eg. $121 = (12 - 1)^2 = 121$). Derived from the equation $X^2 - (2C + 10^n)X + (C^2 - C) = 0$ with specific values for n and C, we can see that the numbers such as 121, 10201, 1002001, 6084, 1162084, 82369, 1656369, 132496, 1860496 etc. can be found satisfying the desired condition.

$$x^2 - x(2c + 10^n) + c^2 - c = 0$$

For example, when $c = 1$, $c^2 - c$ will be zero and n will be one.

$$\begin{aligned}(x-1)^2 &= 10\times x+1\\ x^2-2x+1 &= 10\times x+1\\ \text{therefore } x^2-x(2+10) &= 0\\ \text{therefore } x^2-12x &= 0\end{aligned}$$

So, $x=0$ or $x=12$

121	$=(12-1)^2$	$=11^2$	$=121$
10201	$=(102-01)^2$	$=101^2$	$=10201$
1002001	$=(1002-001)^2$	$=1001^2$	$=1002001$etc

When $c>1$ and the discriminant is >0 and a perfect square, two values of x are obtained. We get such type of numbers. They are

6084	$=(6-084)^2$	$=(-78)^2$	$=6084$
1162084	$=(1162-084)^2$	$=(1078)^2$	$=1162084$
82369	$=(82-369)^2$	$=(-287)^2$	$=82369$
1656369	$=(1656-369)^2$	$=(1287)^2$	$=1656369$
132496	$=(132-496)$	$=(-364)^2$	$=132496$
1860496	$=(1860-496)$	$=(1364)^2$	$=1860496$

All the positive square numbers are multiples of 11.

If X_1 and X_2 are the two values of $X_1X_2=c^2-c$, we find the following properties in the above pairs

$$\begin{aligned}6\times 1162 &= 84^2-84=84\times 83=6972\\ 82\times 1656 &= 369^2-369=369\times 368=135792\\ 132\times 1860 &= 496^2-496=496\times 495=245520\end{aligned}$$

Products of square numbers in each pair give

$$\begin{aligned} 78^2 \times 1078^2 &= [+1001 \times -084]^2 \\ -287^2 \times 1287^2 &= [+1001 \times -369]^2 \\ -364^2 \times 1364^2 &= [+1001 \times -496]^2 \end{aligned}$$

Product of each pair $= [(10^n + 1)^2(-c)^2]$.

These numbers have been found by trial and error method as they are random in nature. Even though their property is not like that of Jarasandha, it may be worthwhile to give a deeper thought to such numbers and try to frame a general equation to derive them. Trials are underway to achieve this objective. However the readers are also requested to make an attempt.

Lastly, Bhimasena killed the demon Jarasandha by interchanging the parts left to right and right to left on the advice of Lord Krishna. By changing 81 as 18, $(1+8)^2 = 9^2$ cannot be 18.

Acknowledgement: My sincere thanks are due to Prof. P.V. Arunachalam, Vice Chancellor of Dravidian University, Kuppam for suggesting a fitting title for this article.

On a paper by V.Balakrishnan

Prof. K.S.Padmanabhan

M 73/2, 31st Cross Street, Besantnagar,
Madras - 600 090, India.

In a paper entitled "A Geometrical Problem with A Concealed Pitfall" (The Mathematics Teacher 35(3) 1999,175-178), the author considers the following problem:

Problem: In ΔABC, the angular bisectors of $\angle B$ and $\angle C$ intersect at O and meet the opposite sides at E and F respectively. If $OE = OF$ can you conclude that $AB = AC$?

It is proved that $OE = OF$ implies either $AB = AC$ or $\angle A = 60°$ and he draws three inferences at the end. The object of this note is to point out that his third inference is wrong and to pose an analogous problem for the external bisectors of $\angle B$ and $\angle C$.

The third inference referred to is the following:

If $OE = OF$ and $AB = AC$ then ΔABC is equilateral.

To see the flaw in the above statement, consider any triangle ABC with $AB = AC \neq BC$. In the $\Delta^{les} AOE, AOF, AO$ is common, $\angle AEO = \angle AFO = \frac{3}{2}\alpha$,

where, $\angle B = \angle C = \alpha, \angle EAO = \angle FAO = \frac{1}{2}\angle A.$

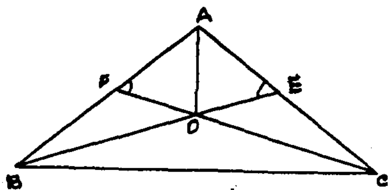


Figure. 1

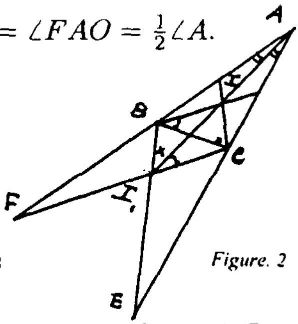


Figure. 2

The two triangles are congruent and so $OE = OF$. However the triangle is not equilateral. We observe that $OE = OF$ is only a necessary but not sufficient condition for $AB = AC$.

It is interesting to investigate the situation involving the external bisectors of $\angle B$ and $\angle C$ in ΔABC.

In this connection we state and prove the following:

Proposition. *In ΔABC, BC is shorter than AB and AC. The external bisectors of $\angle B, \angle C$ meet the opposite sides at E and F. If I_1 is the excentre opposite to A, then $I_1E = I_1F$, if and only if $AB = AC$.*

Proof. The hypotheses $AB > BC$, $AC > BC$ ensure that the external bisectors BI_1, CI_1 meet AC produced and AB produced at E and F respectively. Let I be the incentre of ΔABC. Then IBI_1C is a cyclic quadrilateral and so $\angle AI_1C = \frac{1}{2}\angle B, \angle AI_1B = \frac{1}{2}\angle C$. In $\Delta^{les} AI_1E$, AI_1F we have AI_1 as a common side.

$\angle EAI_1 = \angle FAI_1 = \frac{1}{2}\angle A$. Now let $AB = AC$.

$$\text{Then } \angle AI_1F = \angle BI_1F + \frac{1}{2}\angle C$$

$$\text{and } \angle AI_1E = \angle CI_1E + \frac{1}{2}\angle C = \angle BI_1F + \frac{1}{2}\angle B.$$

Since $\angle B = \angle C$, it follows $\angle AI_1E = \angle AI_1F$. Thus the two triangles are congruent and so $I_1E = I_1F$. Conversely if $I_1E = I_1F$, in the two triangles AI_1E, AI_1F we have AI_1 common, $I_1E = I_1F$ and $\angle I_1AE = \angle I_1AF$. Therefore either the two triangles are congruent or $\angle AEI_1, \angle AFI_1$, are supplementary angles. $\angle AEI_1 = \frac{1}{2}\angle B - \frac{1}{2}\angle A$, $\angle AFI_1 = \frac{\angle C}{2} - \frac{\angle A}{2}$ and their sum is less than $90°$. So the second possibility is ruled out and $\Delta AE_1I_1 \equiv \Delta AFI_1$. This implies $\angle B = \angle C$ or $AB = AC$. The proof is complete.

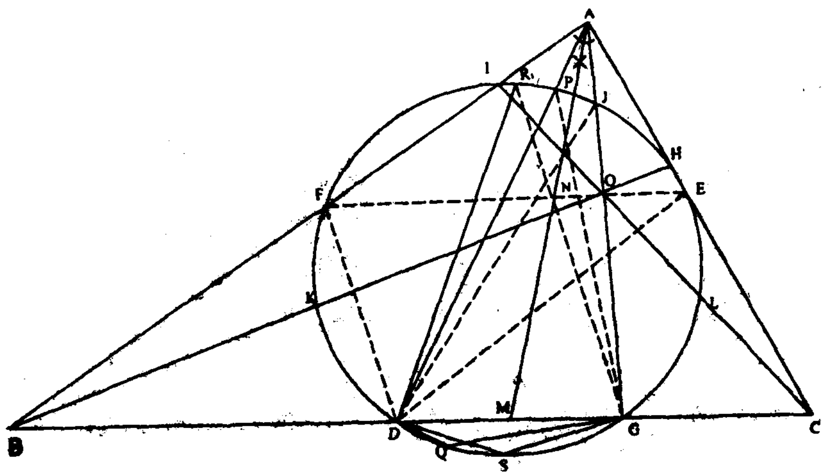
Note: In the above even if BC is greater than AB and AC, the preposition is true and the proof is the same.

WONDER CIRCLE [1]

Manileelan, Suman, Sana

XI Standard, Kendriya Vidyalaya.
AFS Avadi, Chennai.

In connection with a triangle, there are nine special points which lie on the same circle, called nine point circle.



[1] This paper was presented by the authors during Millennium Mathematics Exhibition organized by AMTI in connection with World Mathematical Year, 2000 at Meenakshi College for Women, Chennai on 14^{th} October 2000.

The aim of this paper is to show some salient features of this circle.

1) In fact there are 12 more special points related to the triangle which lie on the same circle. So totally $9 + 12 = 21$ special points lie on this circle. Therefore, we are renaming this circle as 21 point circle.

   While getting the first nine special points of this circle, the altitudes of the triangle play a decisive role i.e., six of the nine points are on the altitudes. But for a triangle medians and angle bisectors are as important as altitudes. It will be unreasonable if they have no contribution to this fantastic circle. Taking this point into account, we prove that there are twelve other special points on the nine point circle contributed by the medians and angle bisectors. So this is named as 21 point circle - a wonder circle.

2) This circle is also the circumcircle of the pedal triangle and also of the mid triangle of ABC.

3) The center of this circle is the mid point of the line segment joining the ortho centre and the circumcentre.

4) The circle is tangential to incircle and all three excircles.

5) The radius of this circle:

   Radius of circumcircle of any $\triangle^{le}$

   $=$ Product of lengths of sides of the triangle divided by four times its area

   The radius of our wonder circle

   $=$ Radius of circumcircle of mid triangle of $\triangle$ ABC

$$= \frac{(\text{Product of its sides } \frac{a}{2}, \frac{b}{2}, \frac{c}{2})}{4(area\triangle ABC/4)}$$
$$= \frac{abc}{8(area\triangle ABC)}$$

6) How did we get 12 more points on this circle? Corresponding to each vertex we get four points. We get P, Q, R and S connected to A as follows.

Using Median:

P: The point of intersection of median from A (AJ) and the perpendicular drawn from G to the same median.

Q: The point of intersection of the perpendiculars drawn at D and J to PD and PJ respectively.

Using angle bisector:

N is the point of intersection of the angle bisector of A i.e., AM and the line joining mid points of the sides AB, AC i.e., EF.

R: The point of intersection of extension of DN and the right bisector of DJ.

S: The point of intersection of the perpendiculars drawn to RD, RJ *at* D and J respectively.

Acknowledgement: We thank Shri. B. Sathyanarayana, PGT Mathematics, Kendriya Vidyalaya, AFS, Avadi for his guidance in the preparation of this paper.

EDITOR'S NOTE: For proof of the results stated in this paper, the authors or the guide may be contacted.

ON SOME CARDBOARD CONSTRUCTIONS

H.S.Keshavamurthy
B1 - 301, white House, 15^{th} cross 6^{th} Main,
RT Nagar, Bangalore 560 032.

In this article, four constructions are given which can be tried by the students in the class room as practical work.

1. To cut a regular ~~hexagon~~ into five parts so that these cut pieces may be represented to form a SQUARE

Construct a regular hexagon whose side is a units, call it $ABCDEFGH$. Let the midpoints of AB, CD, EF and GH be P, Q, R & S. At P, Q, R and S make angles BPJ, DQK, FRL and HSM each equal to $65°28'$. This construction will create 4 identical pentagons $PJQCB$, $QKRED$, $RLSGF$ and $SMPAH$ and a square $JKLM$.

Now the newly discovered square $JKLM$ is held in the same position and the pentagons are repositioned but laterally inverted so that corner A of the pentagon coincides with L of the square and corner B of the pentagon coincides with M of the *square*. Such placements of the three other pentagons, the sides of the square will discover

a BIG square $JKLM$.

Calculation and proof

In the pentagon $SMPAH$, $\hat{A} = \hat{H} = 135°$; $\hat{P} = 114° \ 32'$ (since $A\hat{P}M + B\hat{P}M = 180°$ and $B\hat{P}M = 65°28'$);

$\hat{S} = 65° \ 28'$ (by construction) and

$\hat{M} = 90°$ (since sum of interior angles of pentagon $= 540°$).

Therefore $\angle LMJ = 90°$. Similarly $\angle MJK = \angle JKL = \angle KLM = 90°$.

Therefore $JKLM$ ia a square.

Join SP, GO, FO and NOY

In the isosceles triangle GOF, N is the midpoint of base GF and $GO = OF$ and therefore $ON \perp GF$.

In the right angled triangle GON.

$NGO = 67°30'$ and $GN = \frac{a}{2}$ units.

$NO = GNtan67°30' = \frac{a}{2} \times 2.414 = 1.207a$ units.

$NOY = 2NO = 2 \times 1.207a = 2.414a$ units.

Therefore $GB = NY = 2.414a$ units.

In the isosceles trapezium $HGBA$

$H\hat{S}P = A\hat{P}S = 45°$

$SP = \frac{1}{2}(HA + GB) = \frac{1}{2}(a + 2.414a) = 1.707$a units.

In the triangle SMP,

$SP = 1.7a$ units.

$S\hat{P}M = A\hat{P}M - A\hat{P}S = 114°32' - 45° = 69°32'$.

$SM = SPsin69°32' = 1.599a$ units.

$PM = SPsin20°28' = 0.596a$ units.

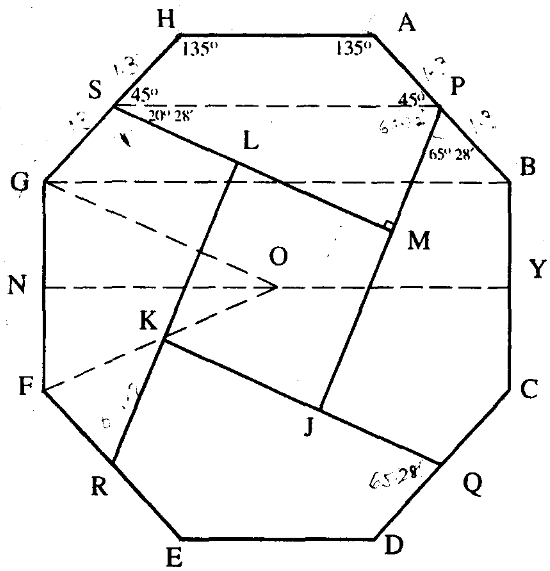


Figure 1: It may be inferred physically or mathematically that $PM = QJ = RK = SL = \frac{\textit{side of the square}-a\text{units}}{2}$ where a is the side of the octagon

By Pythagoras theorem, $SP^2 = SM^2 + MP^2$.

Therefore ΔSMP is right angled at M.

By construction $SL = MP$ and $ML = MS - SL = 1.59a - 0.59a = a$ units (Approximation up to second decimal).

2. To cut a regular Hexagon into five parts so that these cut pieces may be repositioned to form a RHOMBUS

$ABCDEF$ is a regular hexagon whose side is a units and center O. P, Q, R, S, T, V are the midpoints of $FA, AB, \cdots$ and EF. Join PQ, RS, ST and VP. The

triangles QAP, RCS, VFP and TSD are cut out. The triangle QAP is imposed laterally inverted such that the side QA coincides with QB.

$\angle QBX = \angle QAP = 120^\circ$ and $\angle QBO = 60^\circ$. XBO is a straight line. Similarly triangles RCS, VFP and TDS are repositioned as shown in the figure. Therefore the Rhombus $PXSY$ is discovered.

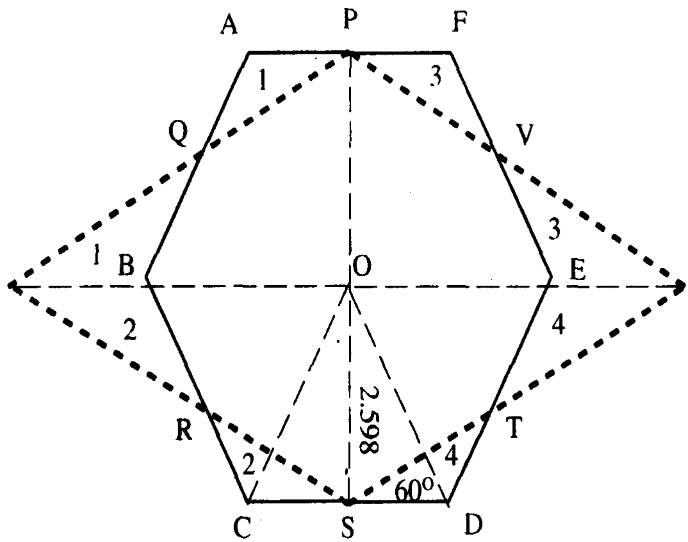


Figure 2:

It may also be proved that the area of Hexagon $ABCDEF$ and Rhombus $PXYS$ is equal to $2.598a^2$ units independently. Thus the hexagon is cut into 5 parts, viz., 4 identical triangles and an octagon and repositioned to form a Rhombus.

3. To cut a regular Hexagon into ~~five~~ parts so that these cut pieces may be repositioned to form a RECTANGLE

$ABCDEF$ is a regular hexagon of side a units. P, Q, R, S and T are the mid points of FA, AB, CD and DE. Join PQ, ST and PD. Thus hexagon is divided into 4 parts, viz., 2 triangles QAP and SDT, 1 trapezium $FPTE$ and 1 hexagon $PQBCST$ and so cut them out.

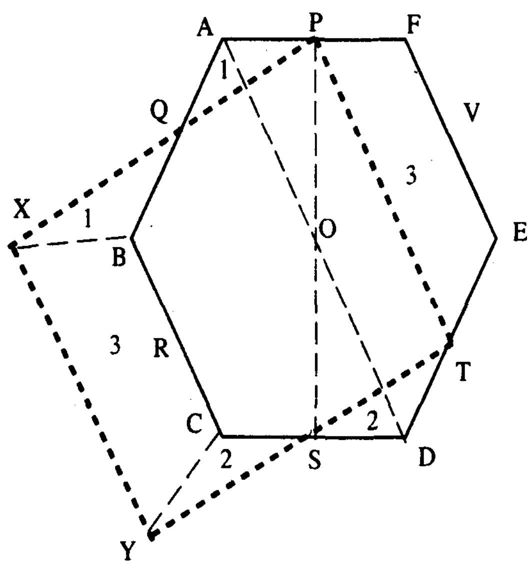


Figure 3:

The triangle QAP is imposed laterally inverted such that the side QA coincides with QB as shown in figure. similarly the triangle SDT is repositioned as SCY. Now place the isosceles trapezium $FPTE$ such that the side FE coincides with BC and the side XY falls on the other side of the hexagon as $BCYX$.

The required rectangle $XYTD$ is thus discovered.

It can be easily proved that the area of Hexagon $ABCDEF$ and that of the rectangle is equal to 2.598 a^2 units independently. Thus the hexagon is cut into 4 parts viz., 2 triangles, 1 trapezium , 1 hexagon and repositioned to form a rectangle.

4. To cut a regular Hexagon into four parts so that these cut pieces may be repositioned to form a RIGHT ANGLED TRIANGLE

$ABCDEF$ is a regular hexagon of side a units. P, Q, R, S, T and V are the mid points of $FA, AB, \cdots EF$ Join PV, PQ, RS and PT and cut along these joined lines to get three identical triangles PFV, QAP and RCS one trapezium $ETPV$ and one Hexagon $PQBRSDT$. These cut pieces are given numbers 1,2,3,4 and 5.

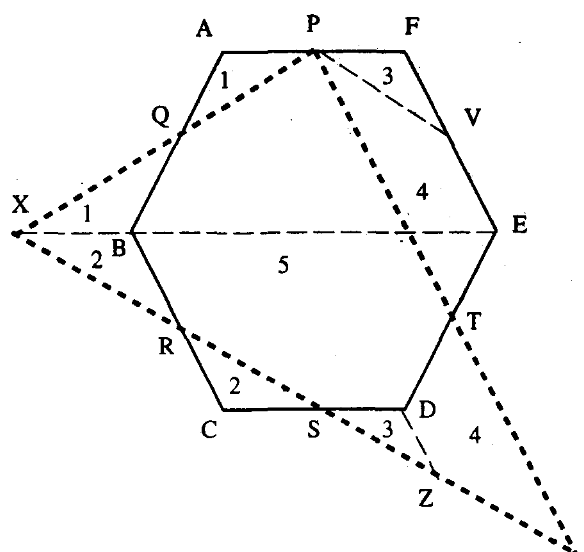


Figure 4:

The three triangles are placed at QBX, RBX and SDZ and the trapezium numbered 4 at $TDZY$. The right angled triangle PXY is discovered.

It may also be proved that area of the hexagon $ABCDEF$ and the right angled triangle is equal to $2.598a^2$ units independently.

Thus the hexagon is cut into five pieces and repositioned to form a right angled triangle.

***

BOOK REVIEW:

Mathemajik Sosity, Bangalore has brought out 2 volumes EXPO - 1 and EXPO - 2 for Mathematics Olympiad in which problems in Algebra, Geometry, Number Theory and Combinatorics are given with solutions. The problems are mostly chosen from the tests conducted by RMO, AMTI, INMO, IMO etc. The books are the outcome of expos conducted for Mathematics Olympiad at the National College, Jayanagar, Bangalore on 10th October, 1999 and 27th August, 2000. Dr. C.R. Pranesachar (Scientific Officer D.A.E. based at Department of Mathematics, Indian Institute of Science, Bangalore), Mr. S.A.Rahim (Director, Mathemajik Sosity, Bangalore) and Mr. N.R.Prasad (First Ranks KRMO and CET 2000 and Medallist IMO, South Korea, 2000 were the resource persons at these Expos. The volumes are priced at Rs.50/- each and can be had from the Director, Mathemajik Sosity, 16/6, K.V. Layout Jayanagar IV Block East, Bangalore - 560011. (Ph: 6343426 and 6652137 - email: mathmaj@vsnl.com - Website: http://education.vsnl.com/mathmaj).

EDITOR, MT.

SOLVING PROBLEM THE ARITHMETICAL WAY

Dr. V. Shankaram

New.# 31 (Old # 15) Purshotham Nagar,
Chrompet, Chennai-45.

There are problems in Arithmetic which require lot of common sense and ingenuity if we are not to use Algebra to solve them. These problems become usual word problems in Algebra in an Algebra class. But the same problems will challenge the best students if treated as problems of Arithmetic. Moreover, it requires a lot of skill to explain the arithmetic solutions to the students. Here we will consider the arithmetical solutions of a few problems. We will also see how these problems act as a good motivation for the students to study Algebra.

Problem 1. A certain sum of money at simple interest amounts to Rs.260/- in 3 years and Rs.300/- in 5 years. Find the sum of money and the rate of simple interest.

Solution. As the amounts at the end of 3 years and 5 years are Rs.260/- and Rs.300/- respectively, the interest for $5-3=2$ years is Rs.(300 - 260)=Rs.40/-. So the interest for one year is Rs.40 $\div$ 2 = Rs.20/-. So the interest for 3 years is (Rs.20/-) $\times$ 3=Rs.60/-. Hence the principal is (the amount after 3 years - the simple interest for 3 years) = Rs.(260-60)=Rs.200/-. Also, the rate of simple interest

per annum $= \frac{20}{2} = 10/-$.
Therefore the sum of money in Rs.200/- and the rate of simple interest is 10%.

Note. If we have to solve this problem with the help of Algebra, we assume that the principal money = P, the interest of Rs.100/- for one year $= i$. We form the equations

$$\begin{aligned} P + 3Pi &= 260 \\ \text{and } P + 5Pi &= 300 \end{aligned}$$

Solving those equations we have P=200 and i=10,

Problem 2. A bag contains 175 coins consisting of one rupee, 50 paise and 25 paise denominations. If the total value of the coins of each denomination is the same, how many coins are there of each denomination and what is the total amount in the bag?

Solution. Here 2 fifty paise coins make one rupee and four 25 paise coins make one rupee. Moreover the value of the coins of each denomination is the same. So the total number of coins in the bag (i.e.,175) must be divisible by 1+2+4 = 7, the quotient $\frac{175}{7} = 25$ giving the number of one rupee coins in the bag. So the number of 50 paise coins $= 35 \times 2 = 70$ and that of 25 paise coins $= 35 \times 4 = 140$.

Note.

(a) The following diagrammatic representation of the bag of coins may prove useful to explain the above

argument to the students.

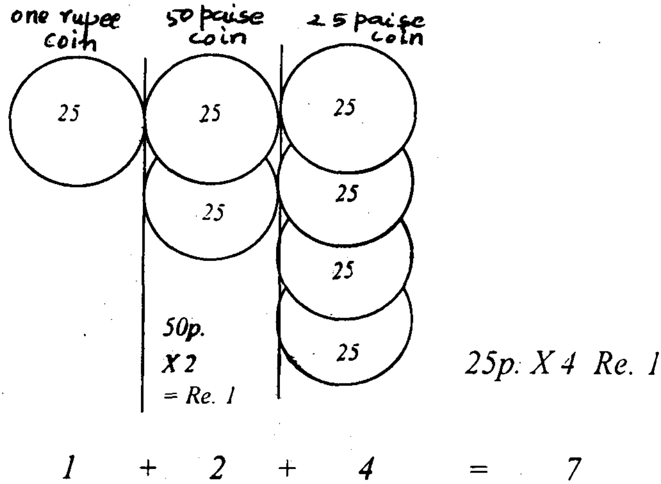


Fig.

(b) To solve this problem by Algebra we suppose that the number of one rupee coins $= x$ and form the equation $x + 2x + 4x = 175$ to have $x = 25$.

Problem 3. After travelling half the distance of a journey by rail and one third of it by car, a man has still 5kms to walk. Find the total distance of his whole journey.

Solution.

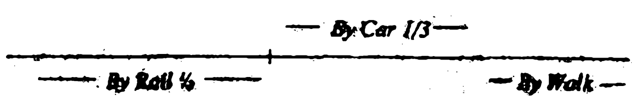


Fig.

The journey covered by rail and car is $\frac{1}{2}+\frac{1}{3}=\frac{3+2}{6}$ $=\frac{5}{6}$ of the total journey. The remaining journey (which is to be covered by walking) is $1-\frac{5}{6}=\frac{1}{6}$ of the total journey. Now according to the problem $\frac{1}{6}$ of the total journey =5kms. (as the journey to be covered by walk is 5kms.) Hence the length of the total journey $= 5 \times 6 = 30kms.$

Note.

(a) Sometimes the teacher supposes that the total journey be 1. This is confusing to the students and wrong. So the language and explanation should be suitably modified with the help of the above diagram.

(b) To solve this problem by Algebra we assume that the length of the total journey $= x$ Kms and frame the equation $x-(\frac{x}{2}+\frac{x}{3})=5$ to arrive at the solution $x=30$.

Problem 4. The product of two numbers is 17496 and the result of dividing the smaller number by the other is $\frac{3}{8}$. Find the numbers.

Solution. Given that

$$\frac{\text{the smaller number}}{\text{the larger number}}=\frac{3}{8}$$

Moreover, 3 is coprime to 8 i.e., there is no common factor of 3 and 8.

So the smaller number is divisible by 3 and the larger number is divisible by 8 and the quotient in each case is the H.C.F. of the two required numbers. As the product of the two numbers is 17496, we have the square of H.C.F. of two numbers

$$= \frac{17496}{3 \times 8} = \frac{2187}{3} = 729 = 27^2$$

(i.e.,) H.C.F $= 27$.
Hence the smaller number $= 27 \times 3 = 81$ and the larger number $= 27 \times 8 = 216$.

Note. To solve this problem let us suppose that the smaller number $= x$ and the other number $= y$ and frame the equations $\frac{x}{y} = \frac{3}{8}$ and $xy = 17496$. The equations imply $x \cdot \frac{(8x)}{3} = 17496 \Rightarrow \frac{8x^2}{3} = 17496 \Rightarrow x = 81$.
So $y = \frac{8 \times 81}{3} = 216$.

All the difficult problems in Arithmetic may not be amenable to arithmetical solutions. For example, in the case of the following problem, the arithmetical solution appears to be too artificial and far-fetched, while its Algebraic solution appears to be very natural. First, we will solve this problem Algebraically and then discuss its arithmetical solution.

Problem 5. A train passes a telegraph post in 8 seconds and a bridge 264 meters long in 20 seconds. Find the length of the train and its speed in kilometers per hour.

Solution. Let x denote the length of the train in meters. Then from the conditions of the problem the speed of the train will be $\frac{x}{8}$ meter/sec. as well as (264+x)/20 meter/sec. So,we have

$$\begin{aligned} \frac{x}{8} &= \frac{264+x}{20} \\ \Rightarrow \frac{x}{2} &= \frac{264+x}{5} \\ \Rightarrow 5x &= 528+2 \Rightarrow 3x = 528 \end{aligned}$$

$$\Rightarrow x = 176 \text{ meters}$$

which is the length of the train. Also, speed of the train $= \frac{x}{8}$ meters/sec. $= \frac{x/1000}{8/3600}$ kmph $= \frac{176 \times 3600}{1000 \times 8}$ kmph $= \frac{-396}{5}$ kmph =79.2 kmph.

Note. Let us try to solve this problem devoid of algebric symbols. Then considering the facts given in the problem we conclude that (the length of the train in meters) ×8 =the speed of the train in kmph, and (the length of the train in meters + 264m) ×20 =the speed of the train in kmph. From these two facts we are constrained to find the speed and the length of the train. This method is most essentially different from the one given above in the solution except that here x is replaced by the phrase 'the length of the train in meters.' So this method of solving the problem (which we may call the arithmetical way) appears to be too artificial and far-fetched in this case.

The following problems (problems.6 to 16) given below are to be tried and examined by the readers whether they

are amenable to arithmetical and algebraic solutions.

Problem 6. A purse contains Rs.41 in 50 paise and 25 paise denominations, the number of coins being 100. Find the number of coins of each denomination.

Problem 7. Divide Rs.61.25 into an equal number of rupees, half rupees and quarter rupees.

Problem 8. I have a certain number of rupees to be distributed among my friends. If I give each friend Rs.30/-, I will be left with Rs.70/-. But if I give each friend Rs.50/-, I will fall short of Rs.30/-. Find the number of my friends and the number of rupees in my possession.

Problem 9. An amount of Rs.4975/- was divided among 150 students so that each boy received Rs 50/- and each girl Rs.25/-. Find the numbers of boys and girls.

Problem 10. Find the least positive integer which when divided by $1\frac{5}{21}$ and $1\frac{5}{7}$, will give an integer as quotient in each case.

Problem 11. Find the least number which is exactly divisible by 7, but when divided by 6,8 and 9 leaves the same remainder in each case.

Problem 12. The average age of Ram, Rahim and Robert is 16 years and that of Rahim, Robert and Rustam is 12 years. If Ram is 15 years old, find the age of Rustam.

Problem 13. 4 goats and 6 cows cost Rs.3800/-, and 5 goats and 7 cows cost Rs.4500/-. Find the cost of a goat and that of a cow.

Problem 14. A man looks at his watch sometime between 2 P.M. and 3 P.M. Mistaking the hour-hand for the minute hand he reads a time which is 57 minutes earlier than the correct time. What is the correct time?

Problem 15. A man's average speed of journey is 32 km.p.h. For $\frac{1}{4}$ of the journey, the speed is 24 km.p.h. What is his average speed for the remaining $\frac{3}{4}$ of the journey.

Problem 16. A clock takes 5 seconds to strike the hour at 5 A.M. How long will it take to strike the hour at 10 A.M.?

SOME DIFFERENTIAL EQUATIONS AND THEIR SOLUTIONS

S.N. Maitra

Dept. of Mathematics, National Defence Academy
Khadakwasla, PUNE - 411 023.

Six differential equations and their solutions are discussed in this paper.

1. Solve the differential equation

$$\frac{d^2y}{dx^2} + y = xsecx.$$

Solution:

Since it is a second order differential equation, it is to be solved by "Variation of parameters" method or by variable separable method which is a primitive one. In order to solve it by the second method, we need to multiply both sides of the equation by a relevant integrating factor which is to be speculated by trial. This is because this differential equation does not admit of any conventional method described in textbooks of differential equations.

Multiplying both sides of the equation by $cosx$ and rearranging the terms, we get

$$\frac{d}{dx}(cosx\frac{dy}{dx}) + sinx\frac{dy}{dx} + ycosx = x$$

or,

$$\frac{d}{dx}(cosx\frac{dy}{dx}) + \frac{d}{dx}(ysinx) = x$$

Integrating both sides, we get

$$cosx\frac{dy}{dx} + ysinx = \frac{x^2}{2} + A \text{ (where A is a constant)}$$

Multiplying by sec^2x, one gets

$$secx\frac{dy}{dx} + ysecx \ \tan x = \frac{x^2}{2}sec^2x + Asec^2x$$

or,

$$\frac{d}{dx}(ysecx) = \frac{x^2}{2}sec^2x + Asec^2x$$

Integrating by parts,

$$ysecx = (A+\frac{x^2}{2})tanx + xlogcosx - \int logcosxdx + B \text{ (constant)}$$

where B is a constant.

2. Solve the differential equation

$$x\frac{dy}{dx} - 3y = \frac{x^6 + y^2}{x^6 + 1}.$$

Solution:

Multiplying both sides of the equation by x^2 and rearranging the terms on the right hand side, we get

$$x^2(x\frac{dy}{dx} - 3y) = x^2 \cdot x^6 \frac{[1 + (\frac{y}{x^3})^2]}{(x^6 + 1)}$$

or

$$\frac{x^3\frac{dy}{dx} - 3yx^2}{(x^3)^2} = x^2\frac{[1 + (\frac{y}{x^3})^2]}{(x^6 + 1)}$$

or

$$\frac{3d(\frac{y}{x^3})}{1 + (\frac{y}{x^3})^2} = \frac{3x^2dx}{(x^3)^2 + 1}$$

Integrating $3tan^{-1}(\frac{y}{x^3}) = tan^{-1}(x^3)+C$ where C is constant

$$\therefore tan^{-1}\frac{3\frac{y}{x^3} - (\frac{y}{x^3})^3}{1 - 3(\frac{y}{x^3})^2} - tan^{-1}(x^3) = C$$

or

$$tan^{-1}\frac{3x^6y - y^3}{x^3(x^6 - 3y^2)} - tan^{-1}x^3 = tan^{-1}k(\text{for some } k)$$

or

$$tan^{-1}\frac{3x^6y - y^3}{x^3(x^6 - 3y^2)} = tan^{-1}\frac{x^3 + k}{1 - kx^3}$$

or

$$y(1 - kx^3)(3x^6 - y^2) = x^3(x^3 + k)(x^6 - 3y^2).$$

3. Solve the differential equation

$$(2x - 1)\frac{d^2y}{dx^2} - (4x^2 + 1)\frac{dy}{dx} + 2(2x^2 - x + 1)y = 0$$

Solution:

Let us take

$$y_1 = \frac{dy}{dx} \text{ and } y_2 = \frac{d^2y}{dx^2};$$

Put $z = y_1 - y$ so that $z_1 = \frac{dz}{dx} = y_2 - y_1$. Then the given equation becomes

$$(2x - 1)(z_1 + y_1) - (4x^2 + 1)(z + y) + 2(2x^2 - x + 1)y = 0$$

or

$$(2x-1)z_1-(4x^2+1)z+(2x-1)y_1-(2x-1)y = 0 \text{ (put } z = y_1-y)$$

or

$$(2x - 1)z_1 - 2xz(2x - 1) - 2z = 0$$

or

$$z_1 - 2xz - \frac{2z}{2x-1} = 0$$

Multiplying both sides by integrating factor e^{-x^2} we get

$$\frac{d}{dx}(ze^{-x^2}) - \frac{2ze^{-x^2}}{2x-1} = 0$$

or,

$$(2x-1)d(ze^{-x^2}) - 2ze^{-x^2}dx = 0$$

Dividing both sides by $(2x-1)^2$ we have by quotient rule

$$\frac{d}{dx}\{\frac{ze^{-x^2}}{2x-1}\} = 0$$

or

$$\frac{ze^{-x^2}}{2x-1} = C \text{ (constant)}$$

Now putting $z = y_1 - y$ and multiplying both sides by integrating factor e^{-x} we have

$$\frac{d}{dx}(ye^{-x}) = C(2x-1)e^{x^2-x}$$

Integrating, we get

$$ye^{-x} = C\int(2x-1)e^{x^2-x}dx + D \text{ (where D is a constant.)}$$

or,

$$ye^{-x} = Ce^{x^2-x} + D$$

or,

$$y = Ce^{x^2} + De^{x}.$$

4. Solve:

$$\frac{d^2y}{dx^2} - y = tanhx$$

Solution:

Multiplying both sides of the equation by $coshx$ we get

$$\frac{d}{dx}(\frac{dy}{dx}coshx) - \frac{d}{dx}(ysinhx) = sinhx.$$

Therefore,

$$\frac{dy}{dx}coshx - ysinhx = coshx + c_1 (\text{where } c_1 \text{ is a constant.})$$

Thus, $$\frac{dy}{dx} - (tanhx)y = 1 + c_1 sechx$$

Multiplying by $sechx$, we get

$$\frac{d}{dx}(ysechx) = sechx + c_1 sech^2 x$$

$$ysechx = \int sechxdx + c_1 \int sech^2 x + c_2 (\text{where } c_2 \text{ is a constant})$$

So, $$ysechx = \int \frac{2}{e^x + e^-x} + c_1 tanhx + c_2$$
or,

$$y = c_1 sinhx + c_2 coshx + 2coshx.tan^{-1}(e^x)$$

5. Solve:

$$x^2 \frac{d^2y}{dx^2} = n(n+1)y$$

Solution:

Multiplying both sides of this equation by x^{n-1} we get

$$x^{n-1}x^2\frac{d^2y}{dx^2} = n(n+1)yx^{n-1}$$

or,

$$\frac{d}{dx}(x^{n+1}\frac{dy}{dx}) - (n+1)x^n\frac{dy}{dx} - n(n+1)yx^{n-1} = 0$$

or

$$\frac{d}{dx}(x^{n+1}\frac{dy}{dx}) - (n+1)\frac{d}{dx}(x^n y) = 0$$

Integrating, we get,

$$x^{n+1}\frac{dy}{dx} - (n+1)x^n y = A \quad \text{(an arbitrary constant.)}$$

Therefore,

$$\frac{dy}{dx} - \frac{(n+1)y}{x} = \frac{A}{x^{n+1}}$$

or

$$\frac{d}{dx}(\frac{y}{x^{n+1}}) = \frac{A}{x^{2n+2}}$$

or

$$\frac{y}{x^{n+1}} = \int \frac{A}{x^{2n+2}}dx + B \quad \text{(where B is an arbitrary constant.)}$$

Therefore

$$\frac{y}{x^{n+1}} = -\frac{A}{(1+2n)x^{2n+1}} + B$$

or

$$y = -\frac{A}{(1+2n)}x^{-n} + Bx^{n+1}$$

6. Solve:

$$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$$

Solution:

Multiplying both sides of the equation by $2\frac{dy}{dx}$; we get

$$2(x^2-1)\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 2x(\frac{dy}{dx})^2 - 2n^2y\frac{dy}{dx} = 0$$

or,

$$\frac{d}{dx}[(x^2-1)y_1^2] - 2n^2y\frac{dy}{dx} = 0 \text{ where } y_1 = \frac{dy}{dx}$$

Integrating,

$$(x^2-1)y_1^2 - n^2y^2 = n^2A^2,$$

(where A is an arbitrary constant).

Then,

$$(x^2-1)y_1^2 = n^2(y^2+A^2)$$

or,

$$\frac{dy}{\sqrt{y^2+A^2}} = \pm\frac{ndx}{\sqrt{x^2-1}}$$

Integrating,

$$log(y+\sqrt{y^2+A^2}) = \pm n log(x+\sqrt{x^2-1}) + logC,$$

(where C is an arbitrary constant.)

$$\therefore y+\sqrt{y^2+A^2} = C(x+\sqrt{x^2-1})^{\pm n}$$

Now, $(\sqrt{y^2+A^2})^2 - y^2 = A^2$

Dividing this equation by the previous one, we get

$$\sqrt{y^2+A^2} - y = \frac{A^2}{C}(x+\sqrt{x^2-1})^{\mp n}$$

Subtracting this equation from the previous one, we get one,

$$y = C(x+\sqrt{x^2-1})^{\pm n} - \frac{A^2}{C}(x+\sqrt{x^2-1})^{\mp n}$$

i.e.

$$y = A(x+\sqrt{x^2-1})^n + B(x+\sqrt{x^2-1})^{-n}$$

where A and B are arbitrary constants.

THIRTY SIXTH ANNUAL CONFERENCE - A REPORT

The thirty sixth annual conference of AMTI was held at Cochin for three days from 27th December to 29th December 2001. The venue was Bhavan's Vidya Mandir, Elamakkara, Cochin-628026. Thanks to the untiring efforts of the local secretary Dr.A.Vijayakumar, Head, Department of Mathematics, Cochin University of Science and Technology (CUSAT) and his devoted team, the conference was a great success. Nearly two hundred delegates registered at the conference.

At 10.00 AM on 27-12-2001, the conference was inaugurated by Dr. Unni Krishnan Nair, Vice-Chancellor of CUSAT. The Chief guest emphasized on the importance of Mathematics in the present day curriculum and made a plea to the teachers of Mathematics to inculcate in the minds of students a love for Mathematics. Earlier, Sri Vijayachandran, General Convenor and Principal, SBOA, Cochin welcomed the gathering. Then, the Secretary, AMTI read the annaul report of the Association and Dr. A.Vijayakumar briefed the delegates on the programme of the conference. Prof. R.C.Gupta delivered the Presidential address on "Some New Studies and Findings regarding Ancient Indian Mathematics". Smt. Meena Viswanathan proposed a vote of thanks.

The academic sessions for the day with "Prof. R.C.Gupta

Endowment Lecture on History of Mathematics" delivered by Prof. George Ghevarghese Joseph, University of Manchester, United Kingdom on the topic 'Enormity of Zero'. Prof. R.C.Gupta chaired the session.

After lunch, the paper presentation session took place. Sri Kameswara Sharma, General Secretary, AIMED, Sri Prabhakar from Guntur, Sri Vangeepuram Sreenivasan and Sri Koteeswara Rao presented papers entitled 'Mathematics Education Made Easy', 'A Glimpse of Finite Elements Analysis' and 'Renovative Mathematical Concepts' respectively. Prof. P.V.Subramanian chaired the session.

Then two students Master Ganesh Babu (X std) assisted by his father Sri C. Halthorai (Mathematics Teacher, Coonoor) and Master Poornam Chander (X std) assisted by Prof. Pattabhiramacharyulu (Warangal) presented their paper on some properties of certain special types of numbers. Smt. K.Shanthi was the Coordinator for this session.

The last session for the day was a Mathemagic show by Prof. S.A.Rahim, Director, Mathemajik Sosity, Bangalore. In the evening one group of interested delegates was taken round the city for sight seeing.

On 28-12-2001, the second day, the conference commenced with Prof. P.L.Bhatnagar Memorial Lecture on Applied Mathematics with Dr. A. Ramakrishnan as chair person. Sri Anand Parthasarathy, I.T. Correspondent, The Hindu, Cochin, (formerly, Scientist, DRDO) gave an instructive and enlightening talk on 'Tapping the Internet:

Online Resources for Mathematics Research and Teaching'. He presented a quick view of the evolution of the personal computer from its initial form to the present stage. He also outlined what types of development can be expected in the use of PC in the near future. He outlined with examples the use of computers in the class room teaching, however cautioning that the role of the teacher and the blackboard could not and should not be ignored.

In the paper presentation session which followed Prof. P.V.Subramanian, Dept. of Mathematics, IIT (Chennai) talked on 'Teaching Differential Equation to Undergraduate Students'. This was followed by an interesting talk by Dr. Pattabhiramacharyulu, Prof. of Mathematics (Retd), R.E.C., Warangal on 'Invariant and Amicable Social Numbers'. Dr. E. Krishnan chaired this session.

Then followed the 'Amateur on Mathematician's Time' with Prof. J.M.Sharma, New Delhi in chair. Sri Vangeepuram Srinivasan (Retd Engineer, Southern Railway), Sri Suryanarayana Moorthy from Guntur and Sri Rangaswamy from Coimbatore spoke on certain recreational aspects of Mathematics.

In the post lunch session Dr. E.Krishnan, Department of Mathematics, University College, Trivandrum delivered 'Prof. Narasinga Rao Memorial Lecture on Methodology on the topic 'Some thoughts on the Teaching of Geometry in Schools' He explained with examples how to make formal learning of Geometry easy and interesting to the students. He interspersed his talk with a lot of historical anecdotes. Sri R. Athmaraman, former Secretary, AMTI

chaired the session.

The panel discussion on 'Ideal Mathematics curriculum at all levels' followed with Sri. R. Athmaraman as Moderator. Smt. Krishnaveni Arunachalam, Retd. Principal, Lady Wellingdon Training College for Women, Chennai, Sri J.M.Sharma, Retd. Principal, Government Schools, Delhi and Dr.E.Krishnan, Trivandrum participated in the discussion. They shared with the delegates their experiences in the framing of curriculum of primary and secondary schools in their respective states and by the NCERT and CBSE.

After the tea break the general body meeting of AMTI took place.

In the evening interested delegates were taken on local trip in two separate groups - one for a boat cruise in the back waters of Kochi and another on a visit to Kalady, the birth place of Adi Sankara.

On the third and the concluding day of the conference, the morning session began with a talk on 'Some Problems in Number Theory' by Prof. V.K.Krishnan, Prof. of Mathematics (Retd), St. Thomas College, Trichur. He explained how to motivate the students according to their mental capabilities and how to spot the talented. He emphasized on introducing the non-routine techniques in problem solving thereby kindling the intuitive and inquisitive attitude in young minds. Prof. P.V. Subramanian chaired this session.

In the open session that followed under the chairmanship

of Prof. R.C.Gupta, Sri Sai Ramakrishna (A.P.), Sri S.A. Rahim (Bangalore), Sri C.Halthorai (Coonoor), Sri Kiran Meghe (Nagpur), Prof. P.V.Subramanian, Sri Vijaya Krishna (Vijayawada), Sri L.C.Reddy (Bangalore) and others shared their views with the fellow delegates. Doing, understanding, memorizing, practising and loving (DUMP LOVE) Mathematics formed key to success according to Sri. Rahim. Various suggestions like interaction among primary, secondary and higher secondary teachers during such conferences was emphasized by the delegates.

The teachers of Mathematics from Kerala, the host State, availed of the opportunity provided by the conference, met separately and formally formed the Kerala Mathematics Teachers Association (KMTA) and elected their office bearers: Dr. V.K.Krishnan - President, Smt. Renuka Menon - Vice President, Smt. Lakshmi Suresh - Secretary and Sri Suresh - Treasurer.

Before lunch, as the concluding programme of the academic activities of the conference, Sri R. Athmaraman assisted by Smt.K.Shanthi conducted a Quiz Programme for the students of the State. After a preliminary round the finals took place. Participants from Arya Central School, Trivandrum were the winners and participants from B.V.M., Elamakkara and St. Joseph's Anglo Indian School, Erur were the runners up at the Junior Level. At the senior level students of Vidyodaya, Therakkal were the winners and students of Arya Central School, Trivandrum and Chinmaya Vidyalaya, Tripunithura were the runners up.

The valedictory function started at 2.00 P.M. Justice K.P.Radhakrishna Menon, Former Judge of Kerala High Court was the Chief Guest. The Honourable Justice exhorted the delegates to preserve the tradition of our country and stressed the importance of an element of sacrifice in teachers of Mathematics and asked them to impart value based education to their students.Prof. R.C.Gupta presided over the function and distributed the prizes to the winners of the Quiz Competition. On behalf of the delegates, Sri V.Srinivasan, Sri J.M. Sharma and Dr. V.K. Krishnan thanked the organizers of the conference. Sri R. Athmaraman and Dr. Vijayakumar proposed vote of thanks of behalf of AMTI and the Local Reception Committee respectively. Thus the thirty sixth annual conference of AMTI at Cochin came to an end.

The Secretary, AMTI wishes to place on record his deep sense of gratitude to the members of the Executive Committee, Dr. Vijayakumar of CUSAT, Sri Vijayachandran, Sri Vasudevan Pillai, the Principal of Bhavan's Vidya Mandir and all the members of the local reception committee for making this conference possible and successful and also for the excellent arrangements made by them for comfortable lodging and for providing boarding facilities for the delegates.

I thank Prof. A.Ramakrishnan and Smt. K.Shanthi for their assistance in the preparation of this report.

M.MAHADEVAN, Secretary, AMTI.

The Association of Mathematics Teachers of India

Office: B 19, Vijay Avenue, 85 V.R.Pillai Street,
Triplicane, Chennai - 600 005. Ph: 844 15 23.

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

(Founded 1965; Registered as S.No. 143 of 1965 under the Societies Registration Act XXI of 1860)

PRICE LIST OF PUBLICATIONS

1. Mathetic Muse - by S.N. Prabhu	Rs. 15/-
2. TAK numbers (Patterns in product sums) -by S.H. Ramasubramanian	Rs. 10/-
3. Gems from the Mathematics Teacher - compiled by S. Muralidharan and G.R.Vijayakumar	Rs. 60/-

BACK VOLUMES

1. Mathematics Teacher : Rs. 40/- for each complete volume.
Rs. 30/- for each Problem issue (up to Vol.36).

Available volumes and issues : 28 only non problem issues, 29, 30, 31, 32, 33, 34, 35, 36 (2000)

2. Junior Mathematician : Rs. 20/- for each complete volume (up to Vol.10)

Available volumes : Vols. 1, 2, 3, 6, 8, 9,10 (2000).

Postage extra

Packing & forwarding charges including postage Rs. 5/- per book.

Orders may be addressed to :

The Treasurer, AMTI
B 19, Vijay Avenue, 85, V.R. Pillai Street,
Triplicane, Chennai - 600 005.

Discount : 10% for single copies for life members.
10% for book- sellers for orders exceeding Rs. 300/-
20% commission for 50 or more subscriptions for Junior Mathematician (JM)

Price FOR COUNTRIES OTHER THAN INDIA SAME FIGURES IN US DOLLARS i.e. instead of rupees read U.S. dollars.